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THEORETICAL MANUAL FOR SEALOADS VERSION 1: A PROGRAM FOR PREDICTION OF NONLINEAR SHIP MOTIONS, SEA LOADS, AND PRESSURE DISTRIBUTION IN THE TIME DOMAIN

W. Qiu
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Theoretical Manual for SEALOADS Version 1.0:

A Computer Program for Prediction of Nonlinear Ship Motions, Sea Loads and Pressure Distribution in the Time Domain

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ABSTRACT

This report presents the theoretical background for the computer code SEALOADS version 1.0, which computes nonlinear ship motions, sea loads and hydrodynamic pressure distributions over the wetted hull surface in the time domain. The external forces acting on the ship hull include the linearized radiation and diffraction forces, nonlinear Froude-Krylov force, viscous roll damping force, bow-flare slamming force and other forces from rudder, maneuvering, etc. The linearized radiation and diffraction forces are obtained from the impulse response functions, which are solved by using the time-domain Green function and applying a non-impulsive input to the ship. The nonlinear Froude-Krylov force is computed at the instantaneous wetted surface of the hull. A two-dimensional wedge model is applied to compute the bow-flare slamming force. The nonlinear equations of ship motion are solved in the time domain by the fourth-order Runge-Kutta technique.

RÉSUMÉ

Le présent rapport expose le contexte théorique concernant le code machine de la version 1.0 du SEALOADS qui sert à calculer les mouvements non linéaires des navires, les charges marines et les distributions de pression hydrodynamique sur la surface mouillée dans le domaine temps. Les forces externes qui s'exercent sur la coque du navire comprennent les forces linéarisées de rayonnement et de diffraction, la force non linéaire de Froude-Krylov, la force de viscosité de l'amortissement en roulis, la force battante sur la proue et d'autres forces venant du gouvernail, de la manœuvre, etc. Les forces linéarisées de rayonnement et de diffraction sont obtenues des fonctions de réponse impulsionnelles, qui sont résolues à l'aide de la fonction temporelle de Green et en appliquant les données d'entrée non impulsionnelles au navire. La force non linéaire de Froude-Krylov est calculée instantanément à la surface mouillée de la coque. On utilise un modèle cunéiforme bidimensionnel pour calculer la force battante sur la proue évasée. Les équations non linéaires du mouvement du navire sont résolues dans le domaine temps par la technique du quatrième ordre de Runge-Kutta.

Theoretical Manual for SEALOADS version 1: A Computer Program for Prediction of Nonlinear Ship Motions, Sea Loads, and Pressure Distribution in the Time Domain

by

W. Qiu and C.C. Hsiung

EXECUTIVE SUMMARY

Introduction

This report outlines the theoretical models in the FORTRAN program SEALOADS version 1.0 for computing ship motions, bending moments and shear forces on the hull, and hydrodynamic pressure distributions over the wetted hull surface. The computation is done in three-dimensional space in the time domain. Nonlinearities in wave forces, including impulsive forces caused by rapid immersion of the flared bow or bow-bottom slamming, are rationally accounted for. In SEALOADS, the external forces acting on the ship hull include the linearized radiation and diffraction forces, nonlinear Froude-Krylov force, viscous roll damping force, bow-flare slamming force and the forces from rudder, maneuvering, and propeller. The nonlinear Froude-Krylov force is computed at the instantaneous wetted surface of the hull. A two-dimensional wedge model is applied to compute the bow-flare slamming force. The nonlinear equations of ship motion are solved by the fourth-order Runge-Kutta technique. Nonlinear wave loads on ships in severe seas predicted by SEALOADS are useful in determining whether the structure of a Canadian Forces ship could withstand a specified severity of seas with an acceptable risk of failure.

Results

The results show that the computer program SEALOADS provides a practical means for calculating large-amplitude nonlinear loads that are beyond the realm of the conventional linear frequency-domain models.

Significance

The extreme loads are a key factor to be considered in the hull structural analysis, but because they occur very rarely, the characteristics of these loads are still poorly understood. SEALOADS serves as a practical analytical tool for examining them.

Future Plans

Future plans include extensive code validation studies, refinement of the algorithms and/or theoretical models to improve the robustness, development of a more advanced method of calculating bow-flare impact force, application of the non-uniform rational B-splines (NURBS) for representing the ship hull surface. As well, development of a comprehensive analysis tool for the realistic hydroelastic response of the ship girder based on the framework of SEALOADS is envisaged.

Manuel théorique de la version 1.0 de SEALOADS, programme d'ordinateur servant à prévoir les mouvements non linéaires des navires, les charges marines et la distribution de pression dans le domaine temps

W. Qiu et C.C. Hsiung

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Introduction

Ce rapport présente les modèles théoriques du programme en FORTRAN SEALOADS, version 1.0, qui calcule les mouvements des navires, les moments de flexion et les forces de cisaillement sur la coque, ainsi que la distribution de la pression hydrodynamique sur la surface mouillée de la coque. Le calcul s'effectue dans l'espace tridimensionnel et dans le domaine temps. La non-linéarité des forces des vagues, y compris les forces impulsives dues à l'immersion rapide du dévers de l'étrave ou au claquement sur la carène avant, sont considérées de façon rationnelle. Dans SEALOADS, les forces externes exercées sur la coque du navire comprennent les forces linéarisées de rayonnement et de diffraction, la force non linéaire de Froude-Krylov, la force d'amortissement hydraulique du roulis, la force de claquement sur le dévers de l'étrave et les forces du gouvernail, de manoeuvre et des hélices. La force non linéaire de Froude-Krylov est calculée sur la surface mouillée instantanée de la coque. Un modèle angulaire bidimensionnel est appliqué au calcul de la force de claquement sur la carène avant. Les équations non linéaires de mouvement du navire sont résolues par la technique de Runge-Kutta du quatrième ordre. Les prévisions que fournit SEALOADS des charges non linéaires des vagues sur les navires dans une mer agitée sont utiles pour déterminer si la structure d'un navire des Forces canadiennes pourrait résister à un état spécifié d'agitation de la mer en ne présentant que des risques de défaillance acceptables.

Résultats

Les résultats démontrent que le programme d'ordinateur SEALOADS constitue un moyen pratique de calculer les charges non linéaires de forte amplitude que n'englobent pas les modèles classiques à domaine fréquence linéaire.

Portée

Les charges extrêmes constituent un facteur clé pour l'analyse structurale des coques, mais leur très faible occurrence n'a pas encore permis d'en bien comprendre les caractéristiques. SEALOADS représente un outil analytique pratique pour leur examen.

Plans pour l'avenir

On prévoit dans l'avenir effectuer des études poussées de validation de code, améliorer les algorithmes et/ou modèles théoriques afin d'accroître la robustesse, élaborer une méthode perfectionnée permettant de calculer la force de l'impact sur la carène avant et appliquer des courbes B-Spline rationnelles non uniformes (NURBS) pour représenter la surface de la coque des navires. On envisage également la mise au point d'un outil d'analyse global qui puisse déterminer la réponse hydroélastique réaliste de la poutre-navire dans le cadre établi par SEALOADS.

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Nomenclature

α	arbitrary constant to control the non-impulsive input motion
α_1^r	yaw gain
α_2^r	yaw rate gain
β	angle of wave propagation relative to the x-axis (180^0 denotes head seas)
β_w	angle of a wedge
Γ	intersection of the mean wetted surface and the still water plane
$\bar{\gamma}_{jk}$	coefficient of hydrodynamic restoring force in the time domain
$\bar{\gamma}_{jk}^f$	coefficient of the hydrodynamic restoring force of the part of the hull from x_c to FP
Δ	weight displacement
$\delta(t-\tau)$	Dirac δ function
δ^r	rudder angle
δ_0^r	threshold value of yaw beyond which the rudder is activated
δ_e^r	effective rudder angle
$\epsilon_0,\ \epsilon_1$	roll damping coefficient
ε_{ijk}	alternating tensor
η	free surface elevation
$\eta_0(t)$	free surface elevation of the incident wave at the origin of the steady-moving coordinate system

$\bar{\eta}_0(t)$	non-impulsive incident wave at the origin of the steady-moving coordinate system
λ_{44}^{v}	viscous roll damping coefficient corresponding to a frequency of encounter
λ_{BK}	bilge keel roll damping coefficient
λ_E	roll damping coefficient of eddy-making resistance of the hull
λ_F	roll damping coefficient of appendages other than the bilge keel
λ_H	hull frictional roll damping coefficient
$\bar{\lambda}_{jk}$	hydrodynamic damping coefficient in the time domain
$\bar{\lambda}_{jk}^{f}$	damping coefficient of the part of the hull from x_c to FP
$\bar{\lambda}$	aspect ratio of the rudder
$\bar{\mu}_{jk}$	added-mass of the ship in the time domain
$\bar{\mu}_{jk}^{f}$	added mass of the part of the hull from x_c to FP
ρ	fluid density
$\sigma(Q;\tau)$	source strength
$\Phi(x,y,z;t)$	total velocity potential of a point (x,y,z) at time t
Φ^{db}	double-body potential function
$\bar{\Phi}$ and $\bar{\phi}$	velocity potential due to the steady translation
ϕ^{db}	the perturbation potential of the uniform stream
ϕ_D or ϕ_7	diffracted wave potential
ϕ_I	incident wave potential
ϕ_k	radiated wave potential in mode k
ψ_k, $\bar{\chi}_k$, φ_k	potential components due to the input motion
Ω	fluid domain
ω	wave frequency
ω_e	frequency of encounter

A_r	rudder area of a single side
$C_d(x_s)$	sectional cross-flow drag coefficient
d	ship draft
D_p	propeller diameter
d_i	translational displacements in the steady-moving coordinate system
dl_i	length of a segment where the impact occurs
e_1	roll angle defined in the viscous damping computation
e_i	angular displacements of the ship
$F(P,Q;t-\tau)$	wave part of the Green function
F_1^{pt}	propeller thrust
F_4^{pt}	propeller torque
$F_{d_k}^s(t)$	sea loads due to diffracted waves
$F_{fk_k}^s(t)$ and $F_{rs_k}^s(t)$	sea loads caused by the incident waves with respect to the Froude-Krylov forces and the restoring forces
$F_{im_k}^s(t)$	sea loads due to bow-flare impact
$F_{in_k}^s(t)$	inertial component of sea loads
$F_k^s(t)$	total sea loads in kth direction
$F_{r_{jk}}^s(t)$	sea loads due to radiated waves
$F^w(t)$	vertical impact force
$F_i^v(t)$	viscous forces
$F_i^{dif}(t)$	diffraction forces
$F_i^{fk}(t)$	Froude-Krylov forces
$F_i^{im}(t)$	bow flare impact forces
$F_i^{mc}(t)$	miscellaneous forces
$F_i^m(t)$	maneuvering forces
$F_i^{pt}(t)$	propeller thrust

$F_i^{rad}(t)$	radiation forces
$F_i^{re}(t)$	restoring forces
$F_i^{rud}(t)$	rudder forces
$\hat{F}_i$	external forces acting on the ship
g	acceleration of gravity
$G(P,Q;t-\tau)$	transient Green function
$G_0(P,Q)$	Rankine source
GZ	GZ curve
$H(t-\tau)$	Heavside unit step function
$\hat{H}(t)$	roll energy of the ship
I_{22}	mass moment of inertia about x-axis
I_{ij}	moment of inertia matrix
J	advancing velocity coefficient
J_0	Bessel function of order zero
k	wave number (except when used as a subscript)
$K_p^D(x,y,z;t)$	pressure response function due to the diffracted waves
$K_{p_k}^R(x,y,z;t)$	pressure response function due to radiated waves
$K_{d_k}^s(t)$	response function of sea loads due to the diffracted waves
K_Q	torque coefficient of propeller
K_T	thrust coefficient of propeller
$K_{jT}^D(t)$	response function for the diffraction force in the jth direction
$K_{jk}^R(t)$	response function of the ship due to the radiated waves
$K_{r_{jk}}^s(t)$	response function of sea loads due to the radiated waves
l_x	horizontal distances between the pressure point C_{rud} and the centre of gravity

l_z	vertical distances between the pressure point C_{rud} and the centre of gravity
l_i	distance between a segment experiencing slamming and a reference section
m	ship mass
m_f	the mass of the part of the ship between the bow and the cross section x_c
M_i	external bending moments acting on the ship
m_k	m-terms
n_p	revolution per second of propeller
n	unit vector normal to the body surface
N_ϕ	roll damping coefficient
$o_m x_m y_m z_m$	steady-moving coordinate system
$o_s x_s y_s z_s$	ship-fixed coordinate system
$OXYZ$	space-fixed coordinate system
$p(x,y,z;t)$	pressure
$p_d^I(x,y,z;t)$	dynamic pressure due to incident waves
$p_s^I(x,y,z;t)$	static pressure due to incident waves
$p_D(x,y,z;t)$	pressure due to diffracted waves
$p_I(x,y,z;t)$	pressure due to incident waves
$p_R(x,y,z;t)$	pressure due to radiated waves
r and r_1	distance between the source and field points
$\mathbf{r}_g$	the position vector
$R(e_1)$	roll restoring moment
r_{xx}	the radius of gyration about x-axis of the segment forward x_c
$S_b^f(t)$	transient wetted surface forward of x_c
$S_b(t)$	transient wetted surface of the body

$\bar{S}_b$	mean wetted surface of the ship
t	time
$T(x_s)$	local draft of the ship
t_0	initial time
T_{sy}	transformation matrix between the ship-fixed coordinate system and the steady-moving coordinate system
t_p	thrust deduction fraction
U_0	ship forward speed
U_{cx}	longitudinal components of current
U_{cy}	lateral components of current
$\tilde{U}$	instantaneous forward speed (the ship's mean forward speed plus the surge velocity)
V_r	relative flow velocity to the rudder
$\mathbf{V}_s(x, y, z; t)$	velocity of the point (x, y, z) on the body surface
$v_2(x_s, t)$	transverse relative velocity between the hull and the water particle of the local cross-flow
$\mathbf{W}$	vector of the steady flow velocity
x_g^f	the x-coordinate of the center of gravity of the ship segment forward of x_c in the ship-fixed frame
x_c	the x-coordinate in the ship-fixed frame of the section considered for sea loads computation
x_g, y_g, z_g	coordinates of the centre of gravity in the ship-fixed coordinate system
x_i	translational displacements of the ship in the ship-fixed coordinate system
X_{m_i}	ship motions in the steady-moving coordinate system
X_i	ship motions in the ship-fixed coordinate system
(X, Y, Z)	coordinates of a point in the space-fixed frame
(x_m, y_m, z_m)	coordinates of a point in the steady-moving frame

(x_s, y_s, z_s)	coordinates of a point in the ship-fixed frame
$Y's$ and $N's$	maneuvering derivatives
z_w, $\dot{z}_w$, $\ddot{z}_w$	vertical displacement, velocity and acceleration of a wedge

Chapter 1

Introduction

A time-domain computer program, SEALOADS version 1.0, has been developed at the Centre for Marine Vessel Development and Research for the Defence Research Establishment Atlantic (DREA). The initial work was described in an earlier report by Cong, *et al.* (1999). This report presents the theoretical background of SEALOADS version 1.0 for the computation of ship motions, sea loads, and pressure distributions in the time domain. The computation of bow-flare impact forces and viscous damping forces are included. A new method for computation of the time-dependent Green function is presented. A scheme proposed by Chen and Malenica (1996) is included to compute derivatives of the velocity potential and m-terms.

1.1 Literature Review

The prediction of ship motions, sea loads, and hydrodynamic pressure distributions over a ship hull is an essential component of ship design. Strip theory, which has been used as a practical tool, gives unsatisfactory predictions at low frequencies and at high forward speeds. The strip-theory approach is not able to compute the pressure distribution over the hull surface except on sections. Recently high-performance computers have stimulated the development of three-dimensional theories to remove some of the deficiencies of strip theory. Several researchers, such as Chang (1977), Inglis and Price (1982) and Guevel and Bougis (1982), have used the three-dimensional panel method to obtain solutions of ship motions in the frequency domain. The frequency-domain panel method is widely used for the case of a body with zero speed. It encounters difficulties in the case of bodies with forward speed and in extreme seas. The frequency-domain Green function is also complicated and difficult to compute when the forward speed is included. An alternative approach is to formulate the ship motion problem directly in the time domain. When forward speed effects are involved, the Green function employed in the time domain is of a simple form, and requires less

computational effort.

The concept of direct time-domain solutions is based on the work of Finkelstein (1957), Stoker (1957) and Wehausen and Laitone (1960). Cummins (1962) and Ogilvie (1964) first discussed the use of time-domain analysis to solve unsteady ship motion problems. The zero forward speed problem was discussed in detail by Wehausen (1967, 1971).

In the linear time-domain formulations, the time-dependent Green function is used to derive a boundary-integral equation at the mean position of the body surface. Under the assumption of small motion and small-amplitude incident waves, the linearization leads to three individual problems: the steady problem, the radiation problem and the diffraction problem. The radiation and diffraction forces acting on the body are expressed in terms of convolution integrals of the arbitrary motion with impulsive response functions. These methods are due to Liapis and Beck (1985), Beck and Liapis (1987), Beck and King (1989), Beck and Magee (1990) and Lin and Yue (1994).

More and more attention has been paid to directly incorporate the nonlinearity into the time-domain formulations. One extension of the linear time-domain model is to impose the body boundary condition on the instantaneous position of the body. The free surface boundary condition remains linear, so that the Green function can still be applicable. The body-exact problem has been solved with various success. Lin and Yue (1990, 1994) and Magee (1994) presented numerical results for heave and pitch of floating bodies in large-amplitude waves. In their computations, the wetted surface has to be re-panelized at each time step. Obviously, this approach needs intensive computing time and automatic panelization. Huang (1997) combined the exact body-boundary condition with a free-surface condition linearized about the incident wave profile. The weak-scatter assumption used by Huang allows for both body motions and incident waves of large amplitude. Results of these studies show that the application of the exact body boundary conditions can lead to more accurate predictions of large-amplitude motions.

1.2 Summary of Current Work

The approach taken here is to use a linear time-domain model to compute the radiation and diffraction forces. The Froude-Krylov forces are computed on the instantaneous wetted surface. The slamming force is considered by using a two-dimensional wedge model (Payne, 1981). The other nonlinear forces, such as viscous damping forces and maneuvering forces, are also taken into account.

Chapter 2 includes the initial boundary-value problem, the nonlinear equation of ship motion, and the linearized radiation and diffraction problems. Pressure distributions over the wetted surface are discussed in Chapter 3. The computation of Froude-Krylov forces, bow flare impact forces, viscous forces and other nonlinear forces, and the computation of hydrodynamic sea loads are emphasized in Chapter 4.

Chapter 2

Mathematical Formulation

2.1 Equations of Ship Motion

When analyzing the motion of a ship, it is convenient to consider three coordinate systems. As shown in Figure 2.1, $OXYZ$ is a *space-fixed* coordinate system with the OXY plane coinciding with the undisturbed water surface. OZ is vertically upward, and OY is chosen so as to make the coordinate system right-handed. In the *steady-moving* coordinate system, $o_m x_m y_m z_m$, the plane $o_m x_m y_m$ coincides with the calm water surface and $o_m z_m$ is positive upward. The ship has a steady forward speed U_0 in the $o_m x_m$ direction and the $o_m x_m y_m z_m$ system moves at the same speed in the $o_m x_m$ direction. The coordinate system, $o_s x_s y_s z_s$ is fixed on the ship, and o_s, is at the point of intersection of the calm water surface, the longitudinal plane of symmetry and the vertical plane passing through midships. The $o_s x_s y_s$ plane coincides with the undisturbed water surface when the ship is at rest, the $o_s x_s z_s$ plane coincides with the longitudinal plane of symmetry. The axis $o_s x_s$ is towards the bow and $o_s y_s$ to port.

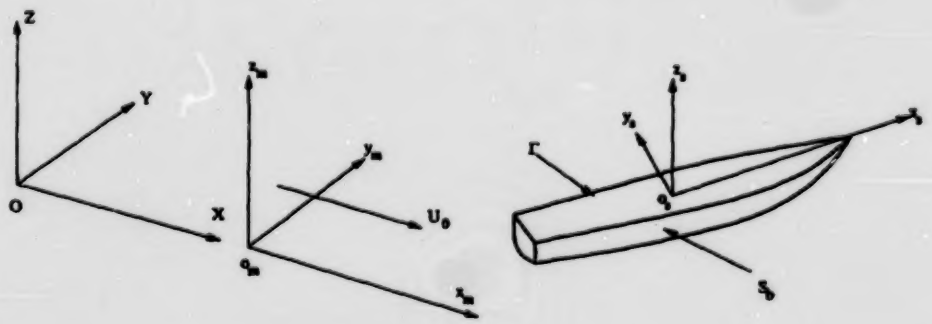

Figure 2.1: The coordinate systems

The oscillatory ship motions are described in the $o_m x_m y_m z_m$ system. The equations of ship motion can be written as

$$m\ddot{x}_i + m\varepsilon_{ijk}\dot{e}_j\dot{x}_k = \bar{F}_i, \quad i = 1,2,3 \tag{2.1}$$

$$I_{ij}\ddot{e}_j + \dot{e}_j\varepsilon_{ijk}I_{kl}\dot{e}_l = M_i, \quad i = 1,2,3 \tag{2.2}$$

where m is the ship mass; I_{ij} is the moment of inertia matrix; x_i denote the translational displacements of the ship in the ship-fixed coordinate system; e_i are the angular displacements, which are the same in the steady-moving and ship-fixed coordinate systems. $\bar{F}_i$ and M_i are the external forces and moments in the ship-fixed system, respectively. The overdots represent differentiation with respect to time; ε_{ijk} can be expressed as

$$\varepsilon_{ijk} = \begin{cases} 0 & \text{if any pair of indices are equal} \\ 1 & \text{if the indices are in cyclic order (123, 231, 312)} \\ -1 & \text{if the indices are in acyclic order (132, 213, 321)} \end{cases} \tag{2.3}$$

By introducing

$$\bar{\mathbf{M}} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{11} & 0 & -I_{13} \\ 0 & 0 & 0 & 0 & I_{22} & 0 \\ 0 & 0 & 0 & -I_{13} & 0 & I_{33} \end{bmatrix} \tag{2.4}$$

$$\dot{\mathbf{X}} = (\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3 \quad \dot{e}_1 \quad \dot{e}_2 \quad \dot{e}_3)^T \tag{2.5}$$

$$\mathbf{F} = (\bar{F}_1 \quad \bar{F}_2 \quad \bar{F}_3 \quad M_1 \quad M_2 \quad M_3)^T \tag{2.6}$$

where $\dot{\mathbf{X}}$ and $\mathbf{F}$ are the velocity vector of ship motions and the external force vector in the ship-fixed frame, respectively. The equations (2.1) and (2.2) can be written as

$$\bar{M}_{ij}\ddot{X}_j + S_i = F_i, \quad i = 1,2,...,6 \tag{2.7}$$

where

$$\mathbf{S} = \left\{ \begin{array}{c} m(\dot{e}_2\dot{x}_3 - \dot{e}_3\dot{x}_2) \\ m(\dot{e}_3\dot{x}_1 - \dot{e}_1\dot{x}_3) \\ m(\dot{e}_1\dot{x}_2 - \dot{e}_2\dot{x}_1) \\ \dot{e}_2\dot{e}_3(I_{33} - I_{22}) - I_{13}\dot{e}_1\dot{e}_2 \\ \dot{e}_1\dot{e}_3(I_{11} - I_{33}) - I_{13}(\dot{e}_3^2 - \dot{e}_1^2) \\ \dot{e}_1\dot{e}_2(I_{22} - I_{11}) + I_{13}\dot{e}_2\dot{e}_3 \end{array} \right\} \tag{2.8}$$

The total external forces acting on the ship are

$$F_i = F_i^{rad} + F_i^{dif} + F_i^{fk} + F_i^{rs} + F_i^{im} + F_i^{v} + F_i^{mc}, \quad i = 1, 2, ..., 6 \tag{2.9}$$

Here, F_i^{rad} and F_i^{dif} are forces due to the radiated and diffracted waves; F_i^{fk} the nonlinear Froude-Krylov forces; F_i^{rs} the restoring forces; F_i^{im} the forces due to the bow-flare impact; F_i^{v} the viscous forces; and F_i^{mc} the miscellaneous forces which include the propeller thrust, F_i^{pt}, maneuvering forces, F_i^{m}, and rudder forces, F_i^{rud}. The computation of these forces are described in the following sections and Chapter 4.

The motions in the steady-moving coordinate system are related to those in the ship-fixed coordinate system as follows:

$$\dot{X}_{m_i} = T_{ij}\dot{X}_j, \quad i = 1, 2, ..., 6 \tag{2.10}$$

$$\mathbf{X}_m = (d_1 \;\; d_2 \;\; d_3 \;\; e_1 \;\; e_2 \;\; e_3)^T \tag{2.11}$$

where $\mathbf{X}_m$ represents the ship motions in the steady-moving coordinate system, and d_i ($i = 1, 2, 3$) are the translational displacements. The transformation matrix $[T_{ij}]$ is

$$[T_{ij}] = \begin{bmatrix} [R_{kl}] & \mathbf{0} \\ \mathbf{0} & [B_{kl}] \end{bmatrix}, \quad i, j = 1, 2, \cdots, 6; \quad k, l = 1, \cdots, 3 \tag{2.12}$$

where

$$[R_{kl}] = \begin{bmatrix} c_2c_3 & s_1s_2c_3 - c_1s_3 & c_1s_2c_3 + s_1s_3 \\ c_2s_3 & s_1s_2s_3 + c_1c_3 & c_1s_2s_3 - s_1c_3 \\ -s_2 & s_1c_2 & c_1c_2 \end{bmatrix} \tag{2.13}$$

and

$$[B_{kl}] = \begin{bmatrix} 1 & s_1t_2 & c_1t_2 \\ 0 & c_1 & -s_1 \\ 0 & s_1/c_2 & c_1/c_2 \end{bmatrix} \tag{2.14}$$

where $c_i = \cos(e_i)$, $s_i = \sin(e_i)$ and $t_i = \tan(e_i)$ for i=1,2 and 3.

Equation (2.7) can be rewritten as

$$\ddot{X}_i = \bar{M}_{ij}^{-1}(F_j - S_j), \quad i = 1, 2, ..., 6 \tag{2.15}$$

Equations (2.9) and (2.15) can be solved by the Runge-Kutta scheme. At every time step, velocities of ship motions in the ship-fixed coordinate system are solved from (2.15), and ship motions in the steady-moving coordinate system are obtained from (2.10).

2.2 The Initial Boundary Value Problem

It is assumed that the fluid is incompressible, inviscid and free of surface tension and that the flow is irrotational. Consider a three-dimensional body in a semi-infinite fluid with a free surface (see Figure 2.1). The ship moves in an incident wave field with a constant speed U_0 and is allowed to perform small unsteady oscillations about its mean positions in six-degrees of freedom.

Under the above assumptions, the fluid velocity of a point (x, y, z) at time t can be described by the gradient of a scalar velocity potential,

$$\mathbf{V}(x, y, z; t) = \nabla\Phi(x, y, z; t), \tag{2.16}$$

Conservation of mass requires that the velocity potential satisfies the Laplace equation in the fluid domain Ω,

$$\nabla^2\Phi = 0 \tag{2.17}$$

The pressure $p(x, y, z; t)$ in the fluid is given by Bernoulli's equation,

$$p(x, y, z; t) = -\rho(\Phi_t + \frac{1}{2}V^2 + gz_0) + p_a \tag{2.18}$$

where g is the acceleration of gravity; ρ is the fluid density; p_a is the atmosphere pressure (constant); Φ_t denotes the differentiation of Φ with time t. By setting $p_a = 0$, the free surface boundary condition may be written as

$$\Phi_{tt} + 2\nabla\Phi \cdot \nabla\Phi_t + \frac{1}{2}\nabla\Phi \cdot \nabla(\nabla\Phi \cdot \nabla\Phi) + g\Phi_{z_0} = 0, \quad \text{on } z_0 = \eta \tag{2.19}$$

where $\eta(x_0, y_0; t)$ is the unknown free surface elevation. Two initial conditions are

$$\left.\begin{array}{l} \Phi = 0 \\ \Phi_t = 0 \end{array}\right\} \quad \text{on } z_0 = 0,\ t < t_0 \tag{2.20}$$

where t_0 is the starting time for the fluid motions which is zero in the radiation problem and $-\infty$ in the diffraction problem.

On the submerged part of the hull, the no flux body-boundary condition must be satisfied,

$$(\nabla\Phi - \mathbf{V}_s) \cdot \mathbf{n} = 0 \quad \text{on } S_b(t) \tag{2.21}$$

where $\mathbf{V}_s(x, y, z; t)$ is the velocity of the point (x, y, z) on the body surface; $\mathbf{n}$ is the unit inner normal vector to the body surface; and $S_b(t)$ is the instantaneous wetted surface of the body. Fluid motions caused by the body will go to zero at spatial infinity for all finite time,

$$\nabla\Phi \to 0, \quad R_1 = \sqrt{x^2 + y^2 + z^2} \to \infty, \quad \text{for } t < \infty \tag{2.22}$$

2.3 Linearization

It is assumed that the fluid disturbances due to the steady forward motion and the unsteady oscillations are small and can be separated. Then both the free surface and the body boundary conditions, as well as the Bernoulli equation can be linearized. The total velocity potential, in the ship-fixed coordinate system can be decomposed as follows:

$$\Phi(x, y, z; t) = \bar{\Phi}(x, y, z) + \bar{\phi}(x, y, z) + \sum_{k=1}^{6} \phi_k(x, y, z; t) + \phi_I(x, y, z; t) + \phi_D(x, y, z; t) \tag{2.23}$$

where the combination of $\bar{\Phi}$ and $\bar{\phi}$ is the velocity potential due to the steady translation for $\bar{\Phi}$ denotes the uniform flow and $\bar{\phi}$ denotes the steady perturbation potential, respectively. This is referred as the *steady* problem. The *radiation* problem is caused by a moving ship which is in prescribed motions of six degrees of freedom with the kth mode, for $k = 1, 2, \cdots, 6$ and the respective wave potential is ϕ_k. The potential of the incident waves is denoted by ϕ_I. For the *diffraction* problem, we denote ϕ_D or ϕ_7 as the potential due to the diffracted waves.

Note that in the steady-moving system, the fluid velocity in the far field will tend to that of the uniform flow and the undisturbed incident wave. Far from the ship, the basis flow must tend to the uniform flow, and this potential is assumed to represent the steady flow around the ship. There are two common choices of basis flow: the uniform flow and the double-body flow.

2.3.1 The Neumann-Kelvin Linearization

The choice of a uniform flow alone, i.e., $\bar{\Phi} = -U_0 x$, leads to the Neumann-Kelvin approximation, where the pressure, the free-surface condition and the body boundary condition are given by:

$$p = -\rho(\phi_t - U_0\phi_x) \tag{2.24}$$

$$(\frac{\partial}{\partial t} - U_0\frac{\partial}{\partial x})^2\phi + g\frac{\partial\phi}{\partial z} = 0, \quad \text{on } z = 0 \tag{2.25}$$

$$\mathbf{n}\cdot\nabla\bar{\phi} = U_0 n_1, \quad \text{on } \bar{S}_b \tag{2.26}$$

$$\mathbf{n}\cdot\nabla(\phi_I + \phi_D) = 0 \quad \text{on } \bar{S}_b \tag{2.27}$$

$$\mathbf{n}\cdot\nabla\phi_k = n_k\dot{x}_k + m_k x_k \quad \text{on } \bar{S}_b \tag{2.28}$$

In equations (2.24) and (2.25), ϕ represents any of the perturbation potentials. $\bar{S}_b$ is the mean position of the ship's surface, and the generalized unit normal n_k is defined as

$$(n_1, n_2, n_3) = \mathbf{n} \tag{2.29}$$

$$(n_4, n_5, n_6) = \mathbf{r}\times\mathbf{n} \tag{2.30}$$

The steady and unsteady potential are coupled through the m-terms in the body boundary conditions. In this case, the m-terms simplify to

$$m_k = (0, 0, 0, 0, U_0 n_3, -U_0 n_2) \tag{2.31}$$

2.3.2 The Double-Body Flow

The Neumann-Kelvin linearization works well for slender bodies. However, for more full-formed bodies, the approximation of the body boundary conditions are not satisfactory. As

an alternative choice to improve the body-boundary condition, the double-body approximation includes a perturbation of the uniform stream. Then, the steady potential is

$$\Phi^{db} = -U_0 x + \phi^{db}, \tag{2.32}$$

which satisfies the following boundary-value problem,

$$\begin{aligned} \nabla^2 \Phi^{db} &= 0 && \\ \frac{\partial \Phi^{db}}{\partial z} &= 0 && \text{on } z = 0 \\ \mathbf{n} \cdot \nabla \Phi^{db} &= 0 && \text{on } \bar{S}_b \\ \nabla \Phi^{db} &\to (-U_0, 0, 0) && \text{as } R_1 \to \infty \end{aligned} \tag{2.33}$$

According to Newman (1978), the double-body m-terms can be written as

$$\begin{aligned} (m_1, m_2, m_3) &= -(\mathbf{n} \cdot \nabla) \nabla \Phi^{db} \\ &= -(n_1 \Phi^{db}_{xx} + n_2 \Phi^{db}_{xy} + n_3 \Phi^{db}_{xz},\ n_1 \Phi^{db}_{yx} + n_2 \Phi^{db}_{yy} + n_3 \Phi^{db}_{yz}, \\ &\quad n_1 \Phi^{db}_{zx} + n_2 \Phi^{db}_{zy} + n_3 \Phi^{db}_{zz}) \end{aligned} \tag{2.34}$$

$$\begin{aligned} (m_4, m_5, m_6) &= \mathbf{r}_g \times (m_1, m_2, m_3) - \mathbf{n} \times \nabla \Phi^{db} \\ &= \mathbf{r}_g \times (m_1, m_2, m_3) + (n_3 \Phi^{db}_y - n_2 \Phi^{db}_z,\ n_1 \Phi^{db}_z - n_3 \Phi^{db}_x, \\ &\quad n_2 \Phi^{db}_x - n_1 \Phi^{db}_y) \end{aligned} \tag{2.35}$$

where $\mathbf{r}_g = (x - x_g, y - y_g, z - z_g)$ is the position vector from the center of gravity (x_g, y_g, z_g) of the ship to a point (x, y, z) on the hull surface. The computation of the double-body m-terms is given in Appendix C.

2.4 The Boundary Integral Equation

The three perturbation problems mentioned above all satisfy the same boundary-value problem with different body boundary conditions. By using the time-dependent Green function the boundary integral equation can be obtained. The time-dependent Green function satisfies the free-surface, far field, radiation and initial conditions. It can be found in Wehausen and Laitone (1960) or Stoker (1957). By defining a field point, $P(x, y, z)$, and a source point, $Q(x', y', z')$, the time-dependent Green function for the infinite water depth can be written as

$$G(P,Q;t-\tau) = -\frac{1}{4\pi}[G_0(P,Q)\delta(t-\tau) + H(t-\tau)F(P,Q;t-\tau)] \tag{2.36}$$

with

$$G_0(P,Q) = \frac{1}{r} - \frac{1}{r_1} \tag{2.37}$$

$$\delta(t-\tau) = \begin{cases} 0 & t \neq \tau \\ \infty & t = \tau \end{cases} \tag{2.38}$$

$H(t-\tau)$ is the Heavside unit step function expressed by

$$H(t-\tau) = \begin{cases} 0 & t < \tau \\ 1 & t \geq \tau \end{cases} \tag{2.39}$$

$$F(P,Q;t-\tau) = 2\int_0^\infty \sqrt{gk}\sin[\sqrt{gk}(t-\tau)]e^{k(z+z')}J_0(kR)dk \tag{2.40}$$

where

$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}, \tag{2.41}$$

$$r_1 = \sqrt{(x-x')^2 + (y-y')^2 + (z+z')^2}, \tag{2.42}$$

$$R = \sqrt{(x-x')^2 + (y-y')^2}, \tag{2.43}$$

and J_0 is the Bessel function of the zeroth order. The computation of the Green function is based on the ordinary differential equation derived by Clement(1998). The analytical solution is given in Appendix A. Applying Green's theorem to the fluid domain Ω, leads to

$$\int\int\int_\Omega (\phi\nabla^2 G - G\nabla^2\phi)d\Omega = \int\int_{S_b} (\phi\frac{\partial G}{\partial n} - G\frac{\partial \phi}{\partial n})dS \tag{2.44}$$

Integrating both side of (2.44) with respect to τ from 0 to t and using the properties of $G(P,Q;t-\tau)$ and the fact that the potential ϕ satisfies the Laplace equation everywhere in the fluid domain, results in the potential in the fluid as

$$\phi(P,t) = -\frac{1}{4\pi}\int_0^t d\tau \int\int_{S_b}(\phi\frac{\partial G}{\partial n} - G\frac{\partial \phi}{\partial n})dS + \frac{1}{4\pi g}\int_0^t d\tau \oint_\Gamma [U_0^2(G\frac{\partial \phi}{\partial x} - \phi\frac{\partial G}{\partial x}) - U_0(G\frac{\partial \phi}{\partial \tau} - \phi\frac{\partial G}{\partial \tau})]dl \quad (2.45)$$

where Γ is the intersection of the mean hull surface and the plane $z = 0$. The positive line integral is in the counter-clockwise direction.

2.4.1 Integral Equation in terms of Source Distribution

The source strength can be applied to avoid spatial derivatives of the potential. Equation (2.45) can be written in the form of a source distribution:

$$\phi(P,t) = -\frac{1}{4\pi}\int_0^t d\tau \int\int_{S_b} G(P,Q;t-\tau)\sigma(Q;\tau)dS - \frac{U_0^2}{4\pi g}\int_0^t d\tau \oint_\Gamma n_1 G(P,Q;t-\tau)\sigma(Q;\tau)dl \quad (2.46)$$

where σ is the source strength which can be solved from

$$\frac{\partial \phi(P;t)}{\partial n} = -\frac{1}{2}\sigma(P;t) - \frac{1}{4\pi}\int_0^t d\tau \int\int_{S_b} \frac{\partial G(P,Q;t-\tau)}{\partial n}\sigma(Q;\tau)dS - \frac{U_0^2}{4\pi g}\int_0^t d\tau \oint_\Gamma [n_1 \frac{\partial G(P,Q;t-\tau)}{\partial n}\sigma(Q;\tau)]dl \quad (2.47)$$

The numerical implementation of (2.47) is presented in Appendix B.

2.5 The Radiation Problem

The radiation problem is subject to the linearized free-surface boundary condition and the body boundary condition. The potential $\phi_k(x,y,z;t)$, for $k = 1, 2, ...6$, satisfies the following equations:

$$\nabla^2\phi_k = 0, \quad \text{in} \quad \Omega \quad (2.48)$$

$$
\begin{aligned}
(\frac{\partial}{\partial t} - U_0\frac{\partial}{\partial z})^2\phi_k + g\frac{\partial \phi_k}{\partial z} = 0, \quad & \text{on} \quad \text{the mean free surface } z = 0 \\
\frac{\partial \phi_k}{\partial n} = n_k\dot{x}_k + m_k x_k, \quad & \text{on} \quad \text{the mean body surface } \bar{S}_b \\
\nabla\phi_k \to 0 \quad & \text{as} \quad R_1 \to \infty, \quad \text{on } z = 0 \\
\nabla\phi_k \to 0 \quad & \text{as} \quad z \to -\infty \\
\phi_k = 0,\ \frac{\partial \phi_k}{\partial t} = 0, \quad & \text{at} \quad t = 0
\end{aligned}
$$

The linearized unsteady pressure is given by

$$p_k(x, y, z; t) = -\rho\frac{\partial \phi_k}{\partial t} - \rho\mathbf{W}\cdot\nabla\phi_k \tag{2.49}$$

where $\mathbf{W} = \nabla(-U_0 x + \phi^{db})$ for the double-body flow approximation.

The hydrodynamic forces acting on the ship can be obtained by integrating the pressure over the wetted surface as follows:

$$F_{jk} = \int\int_{\bar{S}_b} p_k(x, y, z; t) n_j dS = -\rho\int\int_{\bar{S}_b} \frac{\partial \phi_k}{\partial t} n_j dS - \rho\int\int_{\bar{S}_b} (\mathbf{W}\cdot\nabla\phi_k) n_j dS \tag{2.50}$$

The second term of (2.50) involves derivatives of ϕ_k. These can be eliminated by using the following theorem presented in Ogilvie and Tuck (1969) or Ogilvie (1977):

$$\int\int_{\bar{S}_b} [m_j\phi_k + n_j(\mathbf{W}.\nabla\phi_k)] dS = -\oint_\Gamma n_j\phi_k(\mathbf{l}\times\mathbf{n})\cdot\mathbf{W} dl \tag{2.51}$$

where $\mathbf{l}$ is the unit vector tangential to the waterline Γ. Then, the hydrodynamic force can be written as

$$F_{jk} = -\rho\int\int_{\bar{S}_b} \frac{\partial \phi_k}{\partial t} n_j dS + \rho\int\int_{\bar{S}_b} m_j\phi_k dS + \rho\oint_\Gamma n_j\phi_k(\mathbf{l}\times\mathbf{n})\cdot\mathbf{W} dl \tag{2.52}$$

2.5.1 Solution by the Impulse Response Function

Since the initial boundary value problem for radiation has been linearized, the impulse response function can be used to describe the response of the linear system to an arbitrary input. According to the method proposed by Cummins (1962) or Ogilvie (1964), consider that the ship is given an impulsive motion at $t = 0$ in the kth mode with a velocity $\dot{\xi}_k(t) = \delta(t)$. Then, the body boundary condition becomes

$$\frac{\partial \phi_k}{\partial n} = n_k \delta(t) + m_k H(t) \quad \text{for} \quad k = 1, 2, \cdots, 6 \tag{2.53}$$

The body boundary condition (2.53) suggests that the potential ϕ_k can be decomposed into an impulsive and a memory part such that

$$\phi_k(x, y, z; t) = \psi_k(x, y, z)\delta(t) + \chi_k(x, y, z)H(t) \tag{2.54}$$

here, both ψ_k and χ_k satisfy the body boundary condition (2.53) for all time if we set

$$\begin{aligned} \frac{\partial \psi_k}{\partial n} &= n_k \quad \text{on } \bar{S}_b \\ \frac{\partial \chi_k}{\partial n} &= m_k \quad \text{on } \bar{S}_b \end{aligned} \tag{2.55}$$

The ψ_k problem describes the fluid motion during the impulsive stage and satisfies

$$\begin{aligned} \psi_k &= 0 \quad \text{on} \quad z = 0 \\ \frac{\partial \psi_k}{\partial n} &= n_k \quad \text{on} \quad \bar{S}_b \\ \nabla \psi_k &\to 0 \quad \text{at} \quad \infty \end{aligned} \tag{2.56}$$

The χ_k potential represents the motion of the fluid subsequent to the initial impulse and is composed of two components. The first component is because of a change of body orientation due to the impulse in velocity. After the impulse in velocity the body will have a unit displacement in the kth mode. Since the body is in the steady flow field, it results in a change of fluid velocity on the body surface. In order to satisfy the body boundary condition, this change must be canceled. Therefore, $\partial \chi_k / \partial n$ must have the value m_k on the body surface for all $t \geq 0$.

The second component is the result of the impulsive velocity inducing a disturbance into the flow field which will propagate as a wave motion away from the body at subsequent times. Therefore, χ_k will satisfy the free surface condition for $t > 0$ and the initial condition,

$$\begin{aligned} \chi_k &= 0 \quad \text{on} \quad z = 0 \\ \frac{\partial \chi_k}{\partial t} &= -g \frac{\partial \psi_k}{\partial z} \quad \text{on} \quad z = 0 \end{aligned} \tag{2.57}$$

Considering the two components, χ_k can be written as

$$\chi_k = \varphi_k H(t) + \bar{\chi}_k \tag{2.58}$$

where φ_k represents the potential caused by the unit displacement during the impulsive stage of the motion. It satisfies the following boundary conditions:

$$\begin{aligned} \frac{\partial \varphi_k}{\partial n} &= m_k \quad \text{on} \quad \bar{S}_b \\ \varphi_k &= 0 \quad \text{on} \quad z = 0 \\ \nabla \varphi_k &\to 0 \quad \text{at} \quad \infty \end{aligned} \tag{2.59}$$

The potential of the second component $\bar{\chi}_k$ satisfies the following conditions:

$$\begin{aligned} \bar{\chi}_k &= 0 \quad \text{at} \quad t = 0 \\ \frac{\partial \bar{\chi}_k}{\partial t} &= -g \frac{\partial \bar{\chi}_k}{\partial z} \quad \text{on} \quad z = 0 \ \text{ at } t = 0 \\ \frac{\partial^2 \bar{\chi}_k}{\partial t^2} &= -g \frac{\partial \varphi_k}{\partial z} \quad \text{on} \quad z = 0 \ \text{ at } t = 0 \\ \frac{\partial \bar{\chi}_k}{\partial n} &= 0 \quad \text{on} \quad \bar{S}_b \ \text{ for } t \geq 0 \\ [(\frac{\partial}{\partial t} - U_0 \frac{\partial}{\partial x})^2 + g \frac{\partial}{\partial z}](\bar{\chi}_k + \varphi_k) &= 0 \quad \text{on} \quad z = 0 \ \text{ for } t \geq 0 \end{aligned} \tag{2.60}$$

Using (2.58), the potential for an arbitrary forced motion in terms of a velocity , $\dot{\xi}_k(t)$, for the kth mode of motion can be found by integrating (2.54)

$$\begin{aligned} \phi_k(x, y, z; t) &= \int_0^t \phi_k(x, y, z; \tau) \dot{\xi}_k(t - \tau) d\tau \\ &= \psi_k(x, y, z) \dot{\xi}_k(t) + \varphi_k(x, y, z) \xi_k(t) + \int_0^t \bar{\chi}_k(x, y, z; \tau) \dot{\xi}_k(t - \tau) d\tau \end{aligned} \tag{2.61}$$

Here, $\phi_k(x, y, z; t)$ satisfies the body boundary condition, the free surface condition, and the conditions at infinity for all times (Cummins, 1962). Note that ψ_k, φ_k and $\bar{\chi}_k$ can be solved from (2.57), (2.59) and (2.60), respectively.

Substituting Eq(2.61) into (2.52), the radiation force can be expressed as:

$$F_{jk} = -\bar{\mu}_{jk}\ddot{\bar{\xi}}_k(t) - \bar{\lambda}_{jk}\dot{\bar{\xi}}_k(t) - \bar{\gamma}_{jk}\bar{\xi}_k(t) - \int_0^t K^R_{jk}(t-\tau)\dot{\bar{\xi}}_k(\tau)d\tau \tag{2.62}$$

where

$$\begin{aligned}
\bar{\mu}_{jk} &= \rho \int\int_{S_b} \psi_k n_j dS \\
\bar{\lambda}_{jk} &= \rho[\int\int_{S_b} \varphi_k n_j dS - \int\int_{S_b} \psi_k m_j dS \\
&\quad - \oint_\Gamma \psi_k n_j (\mathbf{l}\times\mathbf{n})\cdot\mathbf{W} dl] \\
\bar{\gamma}_{jk} &= -\rho[\int\int_{S_b} \varphi_k m_j dS + \oint_\Gamma \varphi_k n_j (\mathbf{l}\times\mathbf{n})\cdot\mathbf{W} dl] \\
K^R_{jk}(t) &= \rho[\int\int_{S_b} \frac{\partial \bar{\chi}_k}{\partial t} n_j dS - \int\int_{S_b} \bar{\chi}_k m_j dS \\
&\quad - \oint_\Gamma \bar{\chi}_k n_j (\mathbf{l}\times\mathbf{n})\cdot\mathbf{W} dl]
\end{aligned}$$

Here, $\bar{\mu}_{jk}$ is the added-mass of the ship in the time domain which also depends on the ship geometry; $\bar{\lambda}_{jk}$ is the hydrodynamic damping coefficient depending on the ship geometry and the forward speed; $\bar{\gamma}_{jk}$ is the coefficient of the hydrodynamic restoring force in the time domain, which also depends on the ship geometry and forward speed; $K^R_{jk}(t)$ is a function of ship geometry, speed and time and shows the memory effects of the impulse motion.

The impulse response function, K^R_{jk}, can be obtained by using a non-impulsive input velocity:

$$\dot{\bar{\xi}}_k(t) = \sqrt{\frac{\alpha}{\pi}} e^{-\alpha t^2} \tag{2.63}$$

The use of a non-impulsive input can eliminate the high frequency content of input, thus avoiding numerical problems. Here, α is an arbitrary constant which controls the frequency content of the input. It is easily shown that as $\alpha \to \infty$ the input is identical to $\dot{\bar{\xi}}_k = \delta(t)$. The input velocity has the following properties:

$$\bar{\xi}_k(t) = \frac{1}{2}[1 + \mathrm{erf}(\sqrt{\alpha}t] \tag{2.64}$$

$$\ddot{\bar{\xi}}_k(t) = -2\alpha t \dot{\bar{\xi}}_k(t) \tag{2.65}$$

Under this non-impulsive input, the body boundary condition becomes,

$$\frac{\partial \phi_k}{\partial n} = n_k \dot{\xi}_k(t) + m_k \xi_k(t) \tag{2.66}$$

Then, the corresponding $\bar{\mu}_{jk}$, $\bar{\lambda}_{jk}$ and $\bar{\gamma}_{jk}$ can be obtained. Note that the radiation force in (2.52) can also be written as

$$F_{jk}(t) = -\dot{g}_{jk}(t) - h_{jk}(t) \tag{2.67}$$

with

$$g_{jk} = \rho \int\int_{S_b} \phi_k(t) n_j dS \tag{2.68}$$

$$h_{jk} = -\rho \int\int_{S_b} \phi_k(t) m_j dS - \rho \oint_{\Gamma} \phi_k(t) n_j (\mathbf{l} \times \mathbf{n}) \cdot \mathbf{W} dl \tag{2.69}$$

Comparing (2.67) and (2.62) leads to

$$\int_0^t K_{jk}^R(t-\tau)\dot{\xi}_k(\tau)d\tau = \dot{g}_{jk}(t) + h_{jk}(t) - \bar{\mu}_{jk}\ddot{\xi}_k(t) - \bar{\lambda}_{jk}\dot{\xi}_k(t) - \bar{\gamma}_{jk}\xi_k(t) \tag{2.70}$$

where, the impulse response function, K_{jk}^R, is solved for a short duration based on the non-impulsive input $\dot{\xi}_k(t)$. The radiation force due to an arbitrary input motion $\dot{x}_k(t)$ can be obtained from (2.62), where $\dot{x}_k(t)$ is substituted for $\dot{\xi}_k(t)$. The numerical solution of the impulse response function is given in Appendix D.

2.6 The Diffraction Problem

The linearized diffraction problem can be stated as follows:

$$\begin{aligned}
\nabla^2 \phi_7 &= 0 && \text{in } \Omega \\
(\frac{\partial}{\partial t} - U_0 \frac{\partial}{\partial z})^2 \phi_7 + g \frac{\partial \phi_7}{\partial z} &= 0, && \text{on } z = 0 \\
\frac{\partial \phi_7}{\partial n} &= -\frac{\partial \phi_I}{\partial n} && \text{on } \bar{S}_b \\
\nabla \phi_7 &\to 0 && \text{as } R_1 \to \infty, \text{ on } z = 0 \\
\nabla \phi_7 &\to 0 && \text{as } z \to -\infty \\
\phi_7 = 0, \ \frac{\partial \phi_7}{\partial t} &= 0 && \text{as } t \to -\infty
\end{aligned} \tag{2.71}$$

where $\phi_7(x, y, z; t)$ denotes the diffracted wave potential.

2.6.1 Solution Using the Impulse Response Function

Similar to the radiation problem, we can use a non-impulsive incident wave to determine the diffracted wave force. The exciting forces can be computed from

$$F_{j7} = \int_{-\infty}^{\infty} K_{j7}^{D}(t-\tau)\eta_0(\tau)d\tau \tag{2.72}$$

where $K_{j7}^{D}(t)$ is the impulse response function for the diffraction force in the jth mode; $\eta_0(t)$ is the free surface elevation of the incident wave at the origin of the steady-moving coordinate system. The diffraction force can also be expressed as

$$F_{j7}(t) = -\dot{g}_{j7}(t) - h_{j7}(t) \tag{2.73}$$

where

$$g_{j7} = \rho \int\int_{S_b} \phi_7(t) n_j dS \tag{2.74}$$

$$h_{j7} = -\rho \int\int_{S_b} \phi_7(t) m_j dS - \rho \oint_{\Gamma} \phi_7(t) n_j (\mathbf{l} \times \mathbf{n}) \cdot \mathbf{W} dl \tag{2.75}$$

and $\phi_7(t)$ is solved from (2.72). The impulse response function $K_{j7}^{D}(t)$ can be written as

$$\int\int_{-\infty}^{\infty} K_{j7}^{D}(t-\tau)\eta_0(\tau)d\tau = -\dot{g}_{j7}(t) - h_{j7}(t) \tag{2.76}$$

For the case of a ship with forward speed, a non-impulsive incident wave at the origin of the steady-moving reference frame can be chosen as,

$$\bar{\eta}_0(t) = \frac{1}{\pi}\mathrm{Re}\left(\int_0^{\infty} e^{-\omega^2/4\alpha} e^{i\omega_e t} d\omega_e\right) \tag{2.77}$$

where α is an arbitrary constant; $\omega_e = \omega - U_0\omega^2/g\cos\beta$ is the frequency of encounter; β is the angle of wave propagation relative to the x-axis, where 180^0 denotes head seas; ω is the wave frequency. Corresponding to the non-impulsive wave elevation, the derivatives of the incident wave potential can be obtained as

$$\nabla\bar{\phi}_0(x,y,z;t) = \frac{1}{\pi}\mathrm{Re}\left(\begin{Bmatrix} \cos\beta \\ \sin\beta \\ i \end{Bmatrix} \int_0^{\infty} \omega e^{k(z-ix)} e^{-\omega^2/4\alpha} e^{i\omega_e t} d\omega_e\right) \tag{2.78}$$

where $\varpi = x\cos\beta + y\sin\beta$ and $k = \omega^2/g$, the wave number. The details on the original derivation of $\bar{\eta}_0$ and $\nabla\bar{\phi}_0$ were given by King, Beck and Magee (1988).

With the non-impulsive incident wave $\bar{\eta}_0$, the body boundary condition in (2.72) becomes

$$\frac{\partial\phi_7}{\partial n} = -\frac{\partial\bar{\phi}_0}{\partial n} \tag{2.79}$$

The respective $\dot{g}_{j7}(t)$ and h_{j7} can be obtained from (2.74) and (2.75) by solving (2.72). Then, the impulse response function $K_{j7}^D(t)$ can be solved from the following equation:

$$\int\int_{-\infty}^{\infty} K_{j7}^D(t-\tau)\bar{\eta}_0(\tau)d\tau = -\dot{g}_{j7}(t) - h_{j7}(t) \tag{2.80}$$

Chapter 3

Pressure Distribution over the Wetted Hull

The pressure at a point (x, y, z) on the wetted hull can be expressed as

$$p(x, y, z; t) = p_R(x, y, z; t) + p_D(x, y, z; t) + p_I(x, y, z; t) \tag{3.1}$$

where $p_I(x, y, z; t)$ and $p_D(x, y, z; t)$ are the pressures due to the incident waves and the diffracted waves, respectively. The pressure $p_R(x, y, z; t)$ due to the radiated waves is given by

$$p_R(x, y, z; t) = \sum_{k=1}^{6} p_k^R(x, y, z; t) \tag{3.2}$$

3.1 Pressure due to Radiated Waves

For the linearized radiation problem, the pressure at a point (x, y, z) in the kth mode of motion, $p_k^R(x, y, z; t)$, can be related to an input velocity $\dot{x}_k(t)$ by

$$p_k^R(x, y, z; t) = \int_0^t K_{p_k}^R(x, y, z; t - \tau)\dot{x}_k(\tau)d\tau \tag{3.3}$$

where $K_{p_k}^R(x, y, z; t-\tau)$ is the pressure response function due to the motion. It can be solved

by using the non-impulsive input given by (2.63). The pressure corresponding to the non-impulsive input $\dot{\xi}_k$ can be obtained from (2.49). The numerical solution is similar to that of K_{jk}^R (equation 2.70).

3.2 Pressure due to Diffracted Waves

Similar to the solution of the radiation pressure, the pressure due to the diffracted waves can also be defined by a convolution product of the pressure response function, $K_p^D(x,y,z;t)$, and the wave elevation at the origin of the steady-moving system, $\eta_0(t)$, as follows:

$$p_D(x,y,z;t) = \int_{-\infty}^{\infty} K_p^D(x,y,z;t-\tau)\eta_0(\tau)d\tau \tag{3.4}$$

The response function is solved by using the non-impulsive input $\eta_0(t)$ as given in Eq.(2.77).

3.3 Pressure due to Incident Waves

In an incident field, the wave profile $\eta(X,Y;t)$ at any point on the hull surface can be computed in the space-fixed reference frame as follows:

$$p_I(X,Y,Z;t) = p_I^s(X,Y,Z;t) + p_I^d(X,Y,Z;t) \tag{3.5}$$

where the static pressure is

$$p_I^s(X,Y,Z;t) = -\rho g[Z - \eta(X,Y;t)] \tag{3.6}$$

and the dynamic pressure is

$$p_I^d(X,Y,Z;t) = -\rho\frac{\partial\phi_I(X,Y,Z;t)}{\partial t} - \frac{1}{2}\rho|\nabla\phi_I(X,Y,Z;t)|^2 \tag{3.7}$$

and (X,Y,Z) is a point in the space-fixed frame. The elevations and potentials of regular and irregular waves used in the current work are briefly outlined in the corresponding User's guide (Qiu and Hsiung, 1999).

Chapter 4

External Forces and Sea Loads

This chapter describes the computation of the external forces including the Froude-Krylov forces, bow flare impact forces, viscous forces and other nonlinear forces. The computation of the linear radiation and diffraction forces were introduced in Chapter 2. The computational scheme for sea loads at a specific section is also presented.

4.1 Froude-Krylov Forces

At each time step the instantaneous position of any point on the surface of the hull can be computed in the steady-moving coordinate system as follows:

$$\begin{pmatrix} x_m \\ y_m \\ z_m \end{pmatrix} = \begin{pmatrix} d_1 + x_g \\ d_2 + y_g \\ d_3 + z_g \end{pmatrix} + [R_{ij}] \begin{pmatrix} x_s - x_g \\ y_s - y_g \\ z_s - z_g \end{pmatrix}, \quad i,j = 1,2,3 \tag{4.1}$$

where (x_m, y_m, z_m) are the coordinates of a point in the steady-moving coordinate system; (x_g, y_g, z_g) are coordinates of the center of gravity in the ship-fixed coordinate system; (d_1, d_2, d_3) are translational displacements in the steady-moving coordinate system; and (x_s, y_s, z_s) are the coordinates of a point in the ship-fixed coordinate system. $[R_{ij}]$ is the transformation matrix between the ship-fixed coordinate system and the steady-moving coordinate system. The corresponding point in the space-fixed system, (X, Y, Z), can be expressed as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} U_0 t + x_m \\ y_m \\ z_m \end{pmatrix} \tag{4.2}$$

At each time step, the instantaneous position of the hull in the space-fixed system can be obtained from (4.1) and (4.2). The pressure at any point of the hull up to the superstructure can be computed from (3.5).

In the current work, quadrilateral panels are used to discretize the ship's hull. Assuming that the pressure is linearly distributed over a panel, the pressure due to the incident waves at a point on a panel can be expressed as

$$\bar{p}_I(X,Y,Z,t) = \sum_{i=1}^{4} N_i(X,Y,Z) p_{I_i}(X,Y,Z) \tag{4.3}$$

Where N_i is the shape function at node i; p_{I_i} is the pressure at node i. The details of computation are given in Appendix E.

The restoring forces are obtained as

$$F_k^{rs}(t) = -\int\int_{S_b(t)} \bar{p}_I^s n_k dS \tag{4.4}$$

and the nonlinear Froude-Krylov forces are

$$F_k^{fk}(t) = -\int\int_{S_b(t)} \bar{p}_I^d n_k dS \tag{4.5}$$

where $\bar{p}_I^s$ and $\bar{p}_I^d$ are the static and dynamic pressure of a point on a panel, which can be obtained from Equations (3.6), (3.7) and (4.3), respectively.

4.2 Bow Flare Impact

The model for the two-dimensional vertical impact of a wedge in a fluid (Payne, 1981) is applied to the bow flare impact analysis. Assuming the conservation of momentum in a dynamic system, the vertical impact force acting on a wedge of a unit length is expressed as

$$F^w(t) = \frac{\pi \rho z_w f(\beta_w)}{\tan^2 \beta_w} (\dot{z}_w^2 + \frac{1}{2} z_w \ddot{z}_w) \tag{4.6}$$

where $F^w(t)$ is the vertical impact force; z_w, $\dot{z}_w$ and $\ddot{z}_w$ are the vertical displacement, velocity and acceleration of the wedge, respectively; and β_w is the angle of the wedge as illustrated

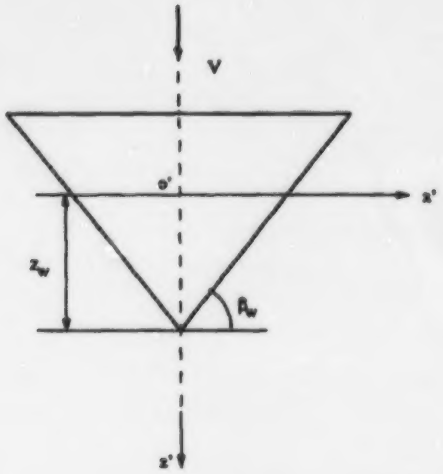

Figure 4.1: An impact wedge model (Payne, 1981)

in Figure 4.1. In the figure, a moving local coordinate system $o'x'z'$ is introduced. The $o'x'$ axis is on the still water surface. The $o'z'$ axis coincides with the centerline of the wedge and is positive upward. The instantaneous position of the wedge angle is used at each time step (Qiu, Cong and Hsiung, 1997).

The vertical shear force and bending moment at a given section is computed as

$$F_3^{im}(t) = \sum_{i=1}^{n} F_i^w(t) dl_i \tag{4.7}$$

$$F_5^{im}(t) = \sum_{i=1}^{n} F_i^w l_i dl_i \tag{4.8}$$

where $F_3^{im}(t)$ and $F_5^{im}(t)$ are the total vertical shear force and bending moment, respectively, due to the bow flare impact; n is the total number of cross section where impact will occur; $F_i^w(t)$ is the vertical slamming force acting on a segment i per unit length; dl_i is the length of the segment and l_i is the distance between a segment experiencing slamming and a reference section. These variables and the force sign conventions are shown in Figure 4.2. Note that the slamming forces are first computed in the space-fixed frame, and then are transferred to the ship-fixed frame.

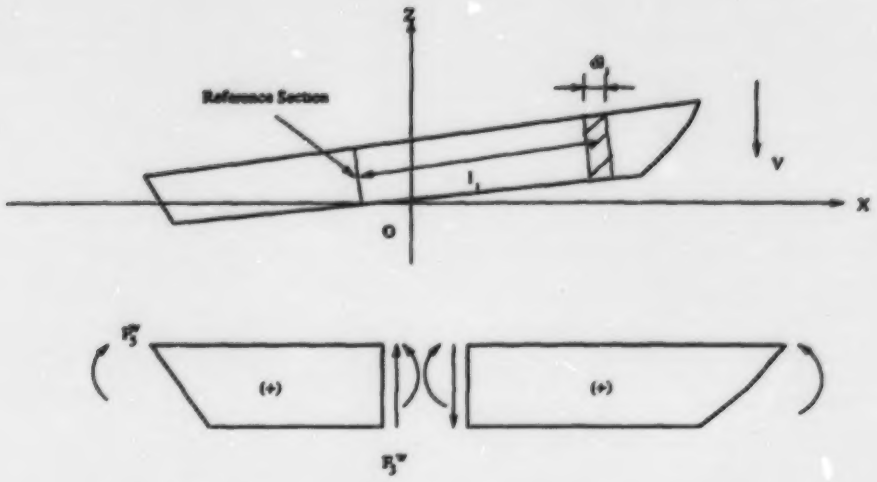

Figure 4.2: Slamming forces and sign conventions

4.3 Viscous Roll Damping Forces

The viscous roll damping forces can be obtained either from empirical formulae or from roll test decay tests.

4.3.1 Roll Damping Coefficients Derived from Roll Decay Test

If roll decay test results are available, the energy method (Haddara and Bennett, 1989) can be used to compute the viscous roll damping coefficient. The energy method states that the rate of change of total energy in the rolling motion is equal to the ratio of energy dissipated by the roll damping. The equation of roll motion, in consistent with equation (2.2), can be written as

$$(I_{22} + \Delta I_{22})\ddot{e_1} + 2N_\phi \dot{e_1} + R(e_1) = 0 \tag{4.9}$$

where e_1 is the roll angle; I_{22} is the mass moment of inertia; ΔI_{22} is the added mass moment of inertia; N_ϕ is the roll damping coefficient. The overdots indicate differentiation with respect to time, and $R(e_1)$ is the restoring moment.

Multiplying both sides of (4.9) by $\dot{e}_1$ and integrating the equation from t_i to t_{i+1}, we obtain

$$\hat{H}(t_i) - \hat{H}(t_{i+1}) = 2\int_{t_i}^{t_{i+1}} N_\phi \dot{e}_1^2 dt \tag{4.10}$$

where $\hat{H}(t)$, the roll energy of the ship, can be written as

$$\hat{H}(t) = \frac{1}{2}(I_{22} + \Delta I_{22})\dot{e}_1^2 + \Delta \int_0^{e_1} GZ(\zeta)d\zeta \tag{4.11}$$

where the last term stands for the potential energy, which is computed from the GZ curve; Δ is the weight displacement. The general form of the damping coefficient is expressed as

$$N_\phi = \epsilon_0 + \epsilon_1|e_1| + \epsilon_2 e_1^2 + \epsilon_3|\dot{e}_1| + \dot{\epsilon}_2 e_1^2 \tag{4.12}$$

Then equation (4.10) is rewritten as

$$\hat{H}(t_i) - \hat{H}(t_{i+1}) = \sum_{j=0}^{4} \alpha_j U_{ij} \tag{4.13}$$

where

$$\begin{aligned}
\alpha_j &= 2\epsilon_j \qquad (4.14)\\
U_{i0} &= \int_{t_i}^{t_{i+1}} \dot{e}_1^2(t)dt\\
U_{i1} &= \int_{t_i}^{t_{i+1}} \dot{e}_1^2(t)|e_1(t)|dt\\
U_{i2} &= \int_{t_i}^{t_{i+1}} \dot{e}_1^2(t)e_1^2(t)dt\\
U_{i3} &= \int_{t_i}^{t_{i+1}} \dot{e}_1^2(t)|\dot{e}_1(t)|dt\\
U_{i4} &= \int_{t_i}^{t_{i+1}} \dot{e}_1^4(t)dt
\end{aligned}$$

The least-square method can be applied to determine the coefficients, α_j, j=0,1,...,4. In fact, the following combination of damping coefficients can be used instead of a general form. Fairly accurate results can be obtained from the following formulae:

Model I: Linear constant plus linear roll

$$N_\phi = \epsilon_0 + \epsilon_1 |e_1| \tag{4.15}$$

Model II: Linear constant plus quadratic roll

$$N_\phi = \epsilon_0 + \epsilon_1 e_1^2 \tag{4.16}$$

Model III: Linear constant plus quadratic roll velocity

$$N_\phi = \epsilon_0 + \epsilon_1 \dot{e}_1^2 \tag{4.17}$$

Model IV: Linear constant plus linear roll velocity

$$N_\phi = \epsilon_0 + \epsilon_1 |\dot{e}_1| \tag{4.18}$$

4.3.2 Estimates of Roll Damping from Empirical Formulae

If there are no roll decay test data available it is assumed that the roll damping coefficient can be computed by

$$N_\phi = \epsilon_0 + \epsilon_1 |e_1| \tag{4.19}$$

The viscous roll damping at a frequency of encounter, ω_e, can be estimated using the work by Schmitke (1978)

$$\lambda_{44}^{\nu}(\omega_e) = \lambda_{BK}(\omega_e) + \lambda_E(\omega_e) + \lambda_H(\omega_e) + \lambda_F(\omega_e) \tag{4.20}$$

where λ_{BK}, λ_E, λ_H, and λ_F denote the contributions from bilge keel, eddy-making resistance of the hull, hull friction and the viscous effect of appendages other than bilge keel at zero speed, respectively. Each component can be estimated from the empirical formulae given by Schmitke (1978). The coefficients, ϵ_0 and ϵ_1 can be obtained by fitting a set of $\lambda_{44}^{\nu}(\omega_e)$ vs. ω_e data. The roll damping moment can be expressed as

$$F_4^{\nu} = \epsilon_0 \dot{e}_1 + \epsilon_1 \dot{e}_1 |e_1| \tag{4.21}$$

4.4 Other Viscous Forces and Moments

4.4.1 Resistance

The resistance and the cross-flow drag are important to surge, sway and yaw. It is recommended that the resistance model data for the hull be used in the computation. If the test data are not available, empirical formulae may be used for the resistance estimation (see Holtrop and Mennen (1982), for example).

4.4.2 Cross-Flow Drag

The cross-flow drag and moment are calculated by

$$F_2^v(t) = -\frac{1}{2}\rho \int_0^L C_d(x_s)T(x_s)v_2(x_s,t)|v_2(x_s,t)|dx_s \tag{4.22}$$

$$F_6^v(t) = -\frac{1}{2}\rho \int_0^L x_s C_d(x_s)T(x_s)v_2(x_s,t)|v_2(x_s,t)|dx_s \tag{4.23}$$

where $C_d(x_s)$ is the sectional cross-flow drag coefficient; $T(x_s)$ is the local draft of the ship; L is the ship length; $v_2(x_s,t)$ is the transverse relative velocity between the hull and the water particle of the local cross-flow.

4.5 Propeller Thrust

The thrust and torque of the propeller are computed by:

$$F_1^{pt} = (1-t_p)\rho n_p^2 D_p^4 K_T(J) \tag{4.24}$$

and

$$F_4^{pt} = \rho n_p^2 D_p^5 K_Q(J) \tag{4.25}$$

where t_p is the thrust deduction fraction, n_p the revolution per second, D_p the propeller diameter, J the advancing coefficient, K_T the thrust coefficient,and K_Q the torque coefficient. The coefficients t_p, J, K_T, and K_Q can be determined from available propeller charts.

4.6 Maneuvering Forces

The following nonlinear lateral force and turning moment are adopted in the seakeeping computation (Kijima et al., 1990):

$$\begin{aligned} F_2^m(t) &= \frac{1}{2}\rho L d\tilde{U}^2(Y_\beta\beta' + Y_r r' + Y_{\beta\beta}|\beta'|\beta' \\ &\quad + Y_{rr}|r'|r' + (Y_{\beta\beta r}\beta' + Y_{\beta rr})\beta' r' \end{aligned} \tag{4.26}$$

and

$$\begin{aligned} F_6^m(t) &= \frac{1}{2}\rho L^2 d\tilde{U}^2(N_\beta\beta' + N_r r' + N_{\beta\beta}|\beta'|\beta' \\ &\quad + N_{rr}|r'|r' + (N_{\beta\beta r}\beta' + N_{\beta rr})\beta' r' \end{aligned} \tag{4.27}$$

where L is the ship's length, d is the ship's draft, $\beta' = -u_2/\tilde{U}$ and $r' = \dot{e}_6 L/\tilde{U}$. In the computation, $\tilde{U}$ is the instantaneous forward speed (the ship's mean forward speed plus the surge velocity),and u_2 the instantaneous sway velocity. The hydrodynamic derivatives $Y's$ and $N's$ are determined by empirical formulae or from test results.

4.7 Rudder Forces and Autopilot

The rudder angle δ^r is defined as being positive when turned to port. The normal force acting on the rudder can be evaluated using relations given by Kijima et al. (1990):

$$F_N = -\frac{1}{2}\rho\frac{6.13\tilde{\lambda}}{\tilde{\lambda}+2.25}A_r V_r^2 \sin\delta_e^r \tag{4.28}$$

where $\tilde{\lambda}$ is the aspect ratio of the rudder, A_r the rudder planform area, V_r the velocity of the water particle relative to the rudder, and δ_e^r the effective rudder angle:

$$\delta_e^r = \delta^r + \tan^{-1}\frac{u_2 - l_z\dot{e}_5 + l_z\dot{e}_4}{\tilde{U}+u_1} \tag{4.29}$$

Suppose that the normal force acts on the center of pressure C_{rud} of the rudder, the rudder force components can be obtained as follows:

$$F_1^{rud} = -|F_N| \sin \delta^r \tag{4.30}$$

$$F_2^{rud} = F_N \cos \delta^r \tag{4.31}$$

$$F_4^{rud} = l_z F_N \cos \delta^r \tag{4.32}$$

and

$$F_6^{rud} = -l_x F_N \cos \delta^r \tag{4.33}$$

where l_z and l_x are horizontal and vertical distances between the center of pressure C_{rud} and the center of gravity, respectively; U_{cx} and U_{cy} are the longitudinal and lateral velocity components of current, respectively.

The following autopilot is considered for course-keeping purposes:

$$\delta^r(t) = \begin{cases} \alpha_1^r e_6(t) + \alpha_2^r \dot{e}_6, & \text{if } e_6 > \delta_0^r \\ 0, & \text{if } e_6 \leq \delta_0^r \end{cases} \tag{4.34}$$

where α_1^r is the yaw gain, α_2^r is the yaw rate gain, and δ_0^r is the threshold value of yaw beyond which the rudder is activated.

4.8 Sea Loads

The hydrodynamic sea loads acting on a cross section x_c can be expressed as

$$F_k^s(t) = \sum_{j=1}^{6} F_{r_{jk}}^s(t) + F_{d_k}^s(t) + F_{fk_k}^s(t) + F_{rs_k}^s(t) + F_{in_k}^s(t) + F_{im_k}^s(t) \tag{4.35}$$
$$for \quad k = 1, 2, \cdots, 6$$

where $F_k^s(t)$ are the total sea loads in the kth mode of motion; $F_{r_{jk}}^s(t)$ and $F_{d_k}^s(t)$ are the sea loads due to the radiated and diffracted waves, respectively; $F_{fk_k}^s(t)$ and $F_{rs_k}^s(t)$ are sea loads caused by the incident waves with respect to the Froude-Krylov forces and the restoring forces; $F_{in_k}^S(t)$ are the sea loads of inertial components; and $F_{im_k}^s(t)$ are the sea loads due to the bow flare impact.

4.8.1 Sea Loads Due to Radiated and Diffracted Waves

The computations of $F^s_{r_{jk}}(t)$ and $F^s_{d_k}(t)$ are similar to the methods used in calculating the radiation and diffraction forces (see Chapter 2). The sea loads are calculated as

$$F^s_{r_{jk}}(t) = -\bar{\mu}^f_{jk}\ddot{x}_k(t) - \bar{\lambda}^f_{jk}\dot{x}_k(t) - \bar{\gamma}^f_{jk}x_k(t) - \int_0^t K^s_{r_{jk}}(t-\tau)\dot{x}_k(\tau)d\tau \tag{4.36}$$

where $\bar{\mu}^f_{jk}$, $\bar{\lambda}^f_{jk}$ and $\bar{\gamma}^f_{jk}$ are the time-domain added mass, damping coefficient, and restoring force coefficient of the ship's segment forward of the section x_c, respectively. The superscript f denotes the segment forward of x_c. $K^s_{r_{jk}}(t-\tau)$ is the impulse response function of sea loads due to the ship motions, which can be obtained from

$$\int_0^t K^s_{r_{jk}}(t-\tau)\dot{\xi}_k(\tau)d\tau = \dot{g}^f_{jk}(t) + h^f_{jk}(t) - \bar{\mu}^f_{jk}\ddot{\xi}_k(t) - \bar{\lambda}^f_{jk}\dot{\xi}_k(t) - \bar{\gamma}^f_{jk}\xi_k(t) \tag{4.37}$$

The sea loads due to the diffracted waves are computed from

$$F^s_{d_k} = \int_{-\infty}^{\infty} K^s_{d_k}(t-\tau)\eta_0(\tau)d\tau \tag{4.38}$$

where $K^s_{d_k}(t-\tau)$ denotes the impulse response function of sea loads due to the diffracted waves and $\eta_0(\tau)$ is the wave elevation at the origin of the steady moving coordinate system, and it can be computed from

$$\int_0^t K^s_{d_k}(t-\tau)\bar{\eta}_0(\tau)d\tau = -\dot{g}^f_{j7}(t) - h^f_{j7}(t) \tag{4.39}$$

4.8.2 Sea Loads Due to Incident Waves

Once the pressure over the panels forward of x_c is determined, the sea loads due to the incident waves can be obtained by integrating the pressure over the wetted surface as

$$F^s_{rs_k}(t) = \int\int_{S^f_b(t)} \bar{p}^s_I n_k dS \tag{4.40}$$

$$F^s_{fk_k}(t) = \int\int_{S^f_b(t)} \bar{p}^d_I n_k dS \tag{4.41}$$

where $S_b^f(t)$ defines the instantaneous wetted surface forward of x_c.

4.8.3 The Inertial Components of Sea Loads

The inertial shear force and bending moment acting on the section x_c are computed as follows:

$$F_{ins}^s(t) = m_f\ddot{x}_3 - m_f(x_g^f - x_g)\ddot{e}_2 \tag{4.42}$$

$$F_{ins}^s(t) = m_f(x_g^f - x_c)\ddot{x}_3 - m_f r_{xx}[r_{xx} - (x_g^f - x_g)]\ddot{e}_2 \tag{4.43}$$

where m_f is the mass of a segment of the ship hull between the bow and the cross-section x_c; x_g^f is the x- coordinate of the center of gravity of the ship segment forward of x_c in the ship-fixed frame; r_{xx} is the radius of gyration about the x-axis of the segment forward x_c.

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Appendix A

Analytical Solution of the Time-Dependant Green Function

The oscillatory part of the time-dependent Green function $F(P,Q;t-\tau)$ given by (2.40) can be analytically solved using an ordinary differential equation derived by Clement (1998).

Eq.(2.40) is rewritten as

$$\bar{F}(\bar{r},\bar{Z};t-\tau)=2\int_0^\infty \sqrt{gk}\sin[\sqrt{gk}(t-\tau)e^{k(z+z')}J_0(k\bar{R})dk \tag{A.1}$$

where $\bar{Z}=\bar{z}+\bar{z}'$. Introducing

$$\begin{aligned} K &= gk \\ z+z' &= \frac{\bar{z}+\bar{z}'}{g} \\ R &= \frac{\bar{R}}{g} \\ F(r,Z;t-\tau) &= g\bar{F}(\bar{r},\bar{Z};t-\tau) \end{aligned} \tag{A.2}$$

leads to

$$F(r,Z;t-\tau)=2\int_0^\infty \sqrt{K}\sin[\sqrt{K}(t-\tau)]e^{K(z+z')}J_0(KR)dK \tag{A.3}$$

By a simple change of variable ($KR_1 \rightarrow \lambda$) in (A.3), the memory part of the time-dependent Green function can be expressed as a function of two real variables (μ,τ):

$$F(r, Z; t) = 2R_1^{-3/2}\tilde{F}(\mu, \tau) \tag{A.4}$$

with

$$\tilde{F}(\mu, \tau) = \int_0^\infty J_0(\lambda\sqrt{1-\mu^2})e^{-\lambda\mu}\sqrt{\lambda}\sin(\sqrt{\lambda}\tau)d\lambda \tag{A.5}$$

where $\mu = -Z/R_1$ and $\tau = t\sqrt{R_1}$. $\tilde{F}(\mu, \tau)$ satisfies the following fourth-order differential equation

$$\tilde{F}^{(4)} + \mu\tau\tilde{F}^{(3)} + (\frac{1}{4}\tau^2 + 4\tau)\tilde{F}^{(2)} + \frac{7}{4}\tau\tilde{F}^{(1)} + \frac{9}{4}\tilde{F} = 0 \tag{A.6}$$

with the initial conditions

$$\tilde{F}^{(2k)}(\mu, 0) = 0, \quad \tilde{F}^{(2k+1)}(\mu, 0) = (-1)^k(k+1)!P_{k+1}(\mu); \quad k = 0, 1, ... \tag{A.7}$$

The derivatives of the Green function, $\partial F/\partial x$ and $\partial F/\partial y$ can be obtained from

$$\frac{\partial F}{\partial x} = \frac{x - x'}{r}F_r \tag{A.8}$$

$$\frac{\partial F}{\partial y} = \frac{y - y'}{r}F_r \tag{A.9}$$

and

$$F_r(r, Z; t) = -2R_1^{-5/2}K(\mu, \tau) \tag{A.10}$$

where $K(\mu, \tau)$ satisfies the fourth-order differential equation:

$$K^{(4)} + \mu\tau K^{(3)} + (\frac{1}{4}\tau^2 + 6\tau)K^{(2)} + \frac{11}{4}\tau K^{(1)} + \frac{21}{4}K = 0 \tag{A.11}$$

with the initial conditions

$$K^{(2k)}(\mu, 0) = 0, \quad K^{(2k+1)}(\mu, 0) = (-1)^k(k+1)!\sqrt{1-\mu^2}\frac{d}{d\mu}P_{k+2}(\mu); \quad k = 0, 1, ... \tag{A.12}$$

Equations (A.6) and (A.11) can be written in a general form:

$$K_F^{(4)} + \mu\tau K_F^{(3)} + (\frac{1}{4}\tau^2 + A\tau)K_F^{(2)} + B\tau K_F^{(1)} + CK_F = 0 \qquad \text{(A.13)}$$

where

$$\begin{aligned} A = 4;\ \ B = \frac{7}{4};\ \ C = \frac{9}{4} \quad &\text{for} \quad K_F = \tilde{F} \\ A = 6;\ \ B = \frac{11}{4};\ \ C = \frac{21}{4} \quad &\text{for} \quad K_F = K \end{aligned} \qquad \text{(A.14)}$$

Now, we can obtain the time-domain Green's function and its derivatives by solving the fourth-order ordinary differential equation (A.13). This equation can be solved by using a Runge-Kutta scheme (Clement, 1998). Here, a series expansion method is applied to achieve an analytical solution (Chuang, 1999).

Assuming

$$K_F(t) = a_0 + \sum_{n=1}^{\infty} a_n (t - t_1)^n \qquad \text{(A.15)}$$

where t_1 is the initial time, then the derivatives of $K_F(t)$ are

$$\begin{aligned} K_F^{(1)}(t) &= a_1 + \sum_{n=2}^{\infty} n a_n (t - t_1)^{n-1} \qquad \text{(A.16)} \\ K_F^{(2)}(t) &= 2a_2 + \sum_{n=3}^{\infty} n(n-1) a_n (t - t_1)^{n-2} \\ K_F^{(3)}(t) &= 6a_3 + \sum_{n=4}^{\infty} n(n-1)(n-2) a_n (t - t_1)^{n-3} \\ K_F^{(4)}(t) &= 24a_4 + \sum_{n=5}^{\infty} n(n-1)(n-2)(n-3) a_n (t - t_1)^{n-4} \end{aligned}$$

If we rewrite the initial conditions as

$$K_F(t_1) = d_0^0,\ \ K_F(t_1) = d_1^0,\ \ K_F(t_1) = d_2^0,\ \ K_F(t_1) = d_3^0 \qquad \text{(A.17)}$$

then

$$a_0 = d_0^0, \quad a_1 = d_1^0, \quad a_2 = d_2^0, \quad a_3 = d_3^0 \tag{A.18}$$

By defining

$$\begin{array}{lll} \kappa_1 = 1, & \kappa_2 = \mu, & \kappa_3 = -\mu t_1 \\ \kappa_4 = \frac{1}{4}, & \kappa_5 = \frac{1}{2}t_1, & \kappa_6 = -\frac{1}{4}t_1^2 + A\mu \\ \kappa_7 = B, & \kappa_8 = Bt_1, & \kappa_9 = C \end{array} \tag{A.19}$$

Eq.(A.13) can be written as

$$\begin{aligned} &\kappa_1 K_F^{(4)} + \kappa_2(t-t_1)K_F^{(3)} + \kappa_3 K_F^{(3)} + \kappa_4(t-t_1)^2 K_F^{(2)} + \\ &\kappa_5(t-t_1)K_F^{(2)} + \kappa_6 K_F^{(2)} + \kappa_7(t-t_1)K_F^{(1)} + \kappa_8 K_F^{(1)} + \kappa_9 K_F = 0 \end{aligned} \tag{A.20}$$

Substituting (A.17) into (A.21) leads to

$$\sum_{i=0}^{9} C_i = 0 \tag{A.21}$$

where

$$\begin{aligned} C_1 &= 24\kappa_1 a_4 + \sum_{n=1}^{\infty} x_1(n+4)(n+3)(n+2)(n+1)a_{n+4}(t-t_1)^n \\ C_2 &= \sum_{n=1}^{\infty} \kappa_2(n+2)(n+1)na_{n+2}(t-t_1)^n \\ C_3 &= 6\kappa_3 a_3 + \sum_{n=1}^{\infty} \kappa_3(n+3)(n+2)(n+1)a_{n+3}(t-t_1)^n \\ C_4 &= \sum_{n=1}^{\infty} \kappa_4 nn(-1)a_n(t-t_1)^n \\ C_5 &= \sum_{n=1}^{\infty} \kappa_5(n+1)na_{n+1}(t-t_1)^n \\ C_6 &= 2\kappa_6 a_2 + \sum_{n=1}^{\infty} \kappa_6(n+2)(n+1)a_{n+2}(t-t_1)^n \end{aligned} \tag{A.22}$$

$$C_7 = \sum_{n=1}^{\infty} \kappa_7 n a_n (t - t_1)^n$$

$$C_8 = \kappa_8 a_1 + \sum_{n=1}^{\infty} \kappa_8 (n+1) a_{n+1} (t - t_1)^n$$

$$C_9 = \kappa_8 a_0 + \sum_{n=1}^{\infty} \kappa_9 a_n (t - t_1)^n$$

Once the coefficients of the series are obtained from (A.21), then we can solve for $\bar{F}(\mu, \tau)$ and $K(\mu, \tau)$. The vertical derivative can be computed from

$$\frac{\partial F}{\partial z} = \frac{\partial F}{\partial Z} = -\frac{\partial^2 F}{\partial t^2} \tag{A.23}$$

Appendix B

Numerical Solution of the Source Strength from the Integral Equation

The boundary integral equation

$$\frac{\partial\phi(P;t)}{\partial n} = -\frac{1}{2}\sigma(P;t) - \frac{1}{4\pi}\int_0^t d\tau \int\int_{S_b} \frac{\partial G(P,Q;t-\tau)}{\partial n}\sigma(Q;\tau)dS - \frac{U_0^2}{4\pi g}\int_0^t d\tau \oint_\Gamma n_1 \frac{\partial G(P,Q;t-\tau)}{\partial n}\sigma(Q;\tau)dl \tag{B.1}$$

Assuming that the source strength is constant over a panel, this equation can be written in the discretized form by using the trapezoidal integration scheme,

$$-\frac{1}{2}\sigma_i^{k_t} + \sum_{j=1}^{n}\sigma_i^{k_t}\mathrm{I}_{ij}^0 + \Delta t[\frac{1}{2}\sum_{j=1}^{n}\sigma_j^0\mathrm{I}_{Fij}^{k_t} + \sum_{k=1}^{k_t-1}\sum_{j=1}^{n}\sigma_j^k\mathrm{I}_{Fij}^{k_t-k}] + \Delta t[\frac{1}{2}\sum_{j=1}^{m}(\sigma_L)_j^0\mathrm{L}_{Fij}^{k_t} + \sum_{k=1}^{k_t-1}\sum_{j=1}^{m}(\sigma_L)_j^k\mathrm{L}_{Fij}^{k_t-k}] = (\mathrm{V}_n)_i^{k_t} \tag{B.2}$$

where i indicates a field point and j indicates a source point; n and m are the number of panels and segments on the waterline, respectively; $\sigma_i^{k_t}$ are the source strengths in six modes of panel i at time step k_t; $\mathbf{V}_n$ is the normal velocities in six modes; $(\sigma_L)_j^{k_t}$ denotes the source strength on waterline segment j at time step k_t; and

$$\mathrm{I}_{ij}^0 = -\frac{1}{4\pi}\iint_{S_{b_j}}(\frac{\partial G_0}{\partial n})_{ij}dS \tag{B.3}$$

$$\mathbf{I}_{Fij}^{k_t-k} = -\frac{1}{4\pi}\iint_{S_j}(\frac{\partial F}{\partial n})_{ij}^{k_t-k}dS \tag{B.4}$$

$$\mathbf{L}_{Fij}^{k_t-k} = -\frac{U^2}{4\pi g}\int_{L_j}(n_1)_j(\frac{\partial F}{\partial n})_{ij}^{k_t-k}dL \tag{B.5}$$

With these definitions, we obtain a system of linear equations which are solved for $\sigma_j^{k_t}$,

$$\sum_{j=1}^{n}\mathbf{A}_{ij}^{k_t}\sigma_j^{k_t} = \mathbf{B}_i^{k_t} \qquad i = 1, 2, \cdots n \tag{B.6}$$

where

$$\mathbf{A}_{ij}^{k_t} = -\frac{1}{2}\delta(i,j) + \mathbf{I}_{ij}^{0} \tag{B.7}$$

and

$$\begin{aligned}\mathbf{B}_i^{k_t} \;=\; & (\mathbf{V}_n)_i^{k_t} - \Delta t[\frac{1}{2}\sum_{j=1}^{n}\sigma_j^0\mathbf{I}_{Fij}^{k_t} + \sum_{k=1}^{k_t-1}\sum_{j=1}^{n}\sigma_j^k\mathbf{I}_{Fij}^{k_t-k}] \\ & -\Delta t[\frac{1}{2}\sum_{j=1}^{m}(\sigma_L)_j^0\mathbf{L}_{Fij}^{k_t} + \sum_{k=1}^{k_t-1}\sum_{j=1}^{m}(\sigma_L)_j^k\mathbf{L}_{Fij}^{k_t-k}]\end{aligned} \tag{B.8}$$

Appendix C

Computation of m-terms

The steady flow effect on the radiation wave problem is shown by the body boundary condition which is represented by m-terms, given in (2.31) and (2.32) as

$$\begin{aligned}(m_1, m_2, m_3) &= -(\mathbf{n}\cdot\nabla)\nabla\Phi^{db} \\ &= -(n_1\Phi^{db}_{xx} + n_2\Phi^{db}_{xy} + n_3\Phi^{db}_{xz},\ n_1\Phi^{db}_{yx} + n_2\Phi^{db}_{yy} + n_3\Phi^{db}_{yz}, \\ &\quad n_1\Phi^{db}_{zx} + n_2\Phi^{db}_{zy} + n_3\Phi^{db}_{zz})\end{aligned}$$

$$\begin{aligned}(m_4, m_5, m_6) &= \mathbf{r}_g \times (m_1, m_2, m_3) - \mathbf{n}\times\nabla\Phi^{db} \\ &= \mathbf{r}_g \times (m_1, m_2, m_3) + (n_3\Phi^{db}_y - n_2\Phi^{db}_z,\ n_1\Phi^{db}_z - n_3\Phi^{db}_x, \\ &\quad n_2\Phi^{db}_x - n_1\Phi^{db}_y)\end{aligned}$$

Since the source strength is assumed to be constant over a panel, it makes the computation of the second-order derivatives of the potential difficult. In the current work, a scheme proposed by Chen and Malenica (1996) is used to compute the derivatives and m-terms. Compared with the scheme used by Huang and Hsiung(1993), the second-order derivatives can be computed individually.

The velocities can be expressed as

$$\frac{\partial\phi^{db}}{\partial x} = \int\int_{S_b} \sigma_x G_0 dS \tag{C.1}$$

$$\frac{\partial\phi^{db}}{\partial y} = \int\int_{S_b} \sigma_y G_0 dS \tag{C.2}$$

$$\frac{\partial \phi^{db}}{\partial z} = \int\int_{S_b} \sigma_z G_0 dS \tag{C.3}$$

where σ_x, σ_y and σ_z are source strengths; Since the velocities are known, the source strengths can be solved from the above equations. The second-order derivatives of the potential can be obtained as

$$\frac{\partial^2 \phi^{db}}{\partial x^2} = \int\int_{S_b} \sigma_x \frac{\partial G_0}{\partial x} dS \tag{C.4}$$

$$\frac{\partial^2 \phi^{db}}{\partial x \partial y} = \int\int_{S_b} \sigma_x \frac{\partial G_0}{\partial y} dS \tag{C.5}$$

$$\frac{\partial^2 \phi^{db}}{\partial x \partial z} = \int\int_{S_b} \sigma_x \frac{\partial G_0}{\partial z} dS \tag{C.6}$$

$$\frac{\partial^2 \phi^{db}}{\partial y^2} = \int\int_{S_b} \sigma_y \frac{\partial G_0}{\partial y} dS \tag{C.7}$$

$$\frac{\partial^2 \phi^{db}}{\partial y \partial z} = \int\int_{S_b} \sigma_y \frac{\partial G_0}{\partial z} dS \tag{C.8}$$

and

$$\frac{\partial^2 \phi^{db}}{\partial z^2} = -\frac{\partial^2 \phi^{db}}{\partial x^2} - \frac{\partial^2 \phi^{db}}{\partial x^2} \tag{C.9}$$

Substituting equations (C.4) to (C.9) into (2.31) and (2.32), the m-terms can be obtained.

Appendix D

Numerical Solution of the Impulse Response Function

As shown in equations (2.70), (2.72), (2.80) and (2.74), the impulse response functions can be solved from the Fredholm integral equation of the first kind. Here, we define a general equation

$$F_r(t) = \int_{t_1}^{t_2} K_r(t-\tau)\eta_I d\tau \tag{D.1}$$

where

$F_r(t) = \dot{g}_{jk}(t) + h_{jk}(t) - \bar{\mu}_{jk}\ddot{\xi}_k(t) - \bar{\lambda}_{jk}\dot{\xi}_k(t) - \bar{\gamma}_{jk}\xi_k(t)$ for the radiation force,
$F_r(t) = -\dot{g}_{j7}(t) - h_{j7}(t)$ for the diffraction force,
$F_r(t) = p_k^R(x,y,z;t)$ for the radiation pressure,
$F_r(t) = p_D(x,y,z;t)$ for the diffraction pressure;

and

$K_r = K_{p_k}^R(x,y,z;t)$, $\eta_I = \dot{\xi}_k(t)$, $t_1 = 0$, $t_2 = t$ for the radiation problem,
$K_r = K_p^D(x,y,z;t)$, $\eta_I = \bar{\eta}_0(t)$, $t_1 = -\infty$, $t_2 = \infty$ for the diffraction problem.

Equation(D.1) can be solved by the direct solution scheme (Cong, Qiu and Hsiung, 1999). The convolution equation can be discretized as

$$F_{r_m} = \sum_{n=1}^{m-1} K_{r_{m-n}}\eta_{I_n}\Delta t + \frac{1}{2}[K_{r_m}\eta_{r_0} + K_{r_0}\eta_{I_m}]\Delta t \quad m = 1,2,...,M \tag{D.2}$$

where M is the total number of steps defined by $t_M = M\Delta t$. With simplified notations,

$$\begin{cases} a_m &= \eta_{r_m} \\ x_m &= K_{r_m} \\ b_m &= F_{r_m} \end{cases}$$

Equation (D.2) becomes a system of equations in terms of x_m,

$$\begin{cases} \frac{1}{2}a_1x_0 + \frac{1}{2}a_0x_1 & & & & & & & = \frac{b_1}{\Delta t} \\ \frac{1}{2}a_2x_0 + a_1x_1 + \frac{1}{2}a_0x_2 & & & & & & & = \frac{b_2}{\Delta t} \\ \cdots & & & & & & & = \cdots \\ \frac{1}{2}a_Mx_0 + a_{M-1}x_1 + \cdots + a_1x_{M-1} + \frac{1}{2}a_0x_M & & & & & & & = \frac{b_M}{\Delta t} \end{cases} \tag{D.3}$$

It is evident that the systems of equations (D.3) are indeterminate, because the total number of unknowns M is larger than the total number of equations by 1. An additional physical condition has to be provided to make (D.3) closed. When time $t_m = m\Delta t$ is sufficiently long, the response function K_{r_m} becomes quite smooth and goes to zero uniformly, i.e, the first-order time derivative of K_{r_m} tends to zero. An additional condition for K_{r_m} at a specific time instant can be obtained by approximating the time derivative with a third-order finite difference, as follows:

$$x_{N-4} - 3x_{N-3} + 3x_{N-2} + x_{N-1} = 0, \quad for \quad N = M + 1 \tag{D.4}$$

Appendix E

Pressure over a Panel

When computing the hydrodynamic pressure due to incident waves, it is assumed that the pressure is linearly distributed over a panel. The hull is discretized by quadrilateral panels. A quadrilateral panel is shown in Fig. E-1.

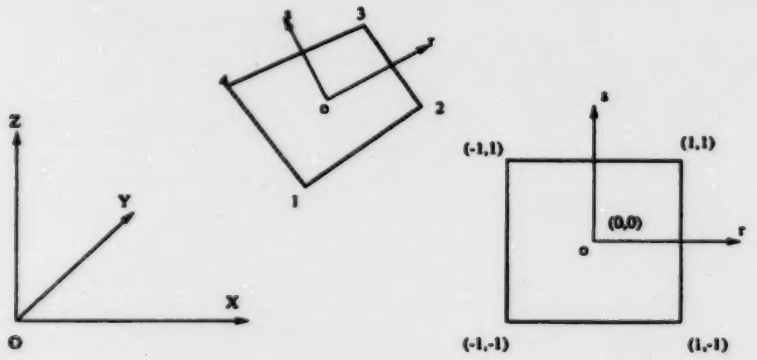

Figure E.1: A quadrilateral panel and the local reference frame

The pressure at any point on a panel due to the incident wave is given by (4.3) as

$$\tilde{p}_I(X,Y,Z;t) = \sum_{i=1}^{4} N_i(X,Y,Z) p_{I_i}(X,Y,Z) \qquad \text{(E.1)}$$

The shape function $N_i(r,s)$ in the local reference frame ors can be expressed as

$$N_1 = \frac{1}{4}(1-r)(1-s)$$

$$
\begin{aligned}
N_2 &= \frac{1}{4}(1+r)(1-s) \\
N_3 &= \frac{1}{4}(1+r)(1+s) \\
N_4 &= \frac{1}{4}(1-r)(1+s)
\end{aligned}
$$

and $r, s \in [-1, 1]$. The pressure at a local coordinates (r, s), is then written as

$$\bar{p}_I(r, s; t) = \sum_{i=1}^{4} N_i(r, s) p_{I_i} \tag{E.2}$$

The Froude-Krylov force, for example, may be obtained as

$$F_k^{Jk}(t) = -\sum_{i=1}^{m}\sum_{j=1}^{n} w_i w_j p_I^d(r_i, s_j) n_k(r_i, s_j) |J(r_i, s_j)| \tag{E.3}$$

where w_i and w_j are the weighting factors at (r_i, s_j), and $|J(r_i, s_j)|$ is the Jacobian determinant.

Appendix F

Code Structures of SEALOADS Version 1.0

The computer program, SEALOADS Version 1.0, for prediction of nonlinear motions, pressure distribution, and sea loads on a ship hull in large waves, consists of four main modules:

(1) radiation module,

(2) diffraction module,

(3) damping module, and

(4) motion, pressure and sea loads module.

The radiation module depends on the forward speed. The impulse response functions for the radiation forces and the pressures on each panel are solved in this module. The diffraction module depends on the forward speed and the wave heading. The impulse response functions for the diffraction forces and the pressures on each panel are solved in this module. Based on the outputs from these two modules, and the damping coefficients from the damping module, the ship motion and pressure distribution and the sea loads due to the radiated and diffracted waves can be computed for the various wave heights in regular and irregular waves. The code structures for SEALOADS Version 1.0 is shown in Figures F.1 to F.4.

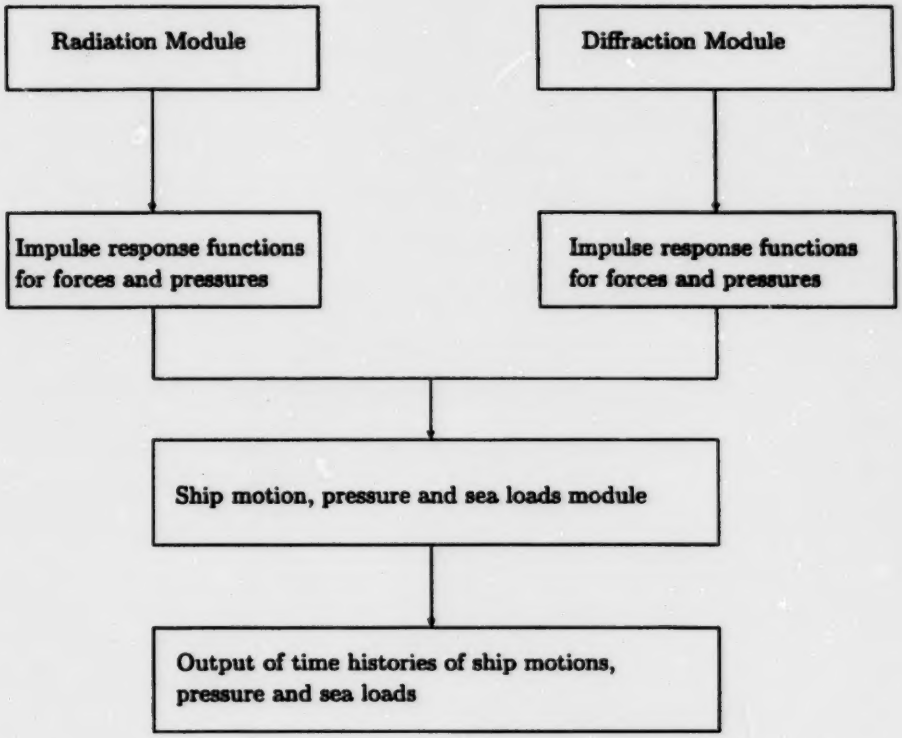
Radiation Module
Diffraction Module
Impulse response functions for forces and pressures
Impulse response functions for forces and pressures
Ship motion, pressure and sea loads module
Output of time histories of ship motions, pressure and sea loads

Figure F.1: Code structure of SEALOADS Version 1.0

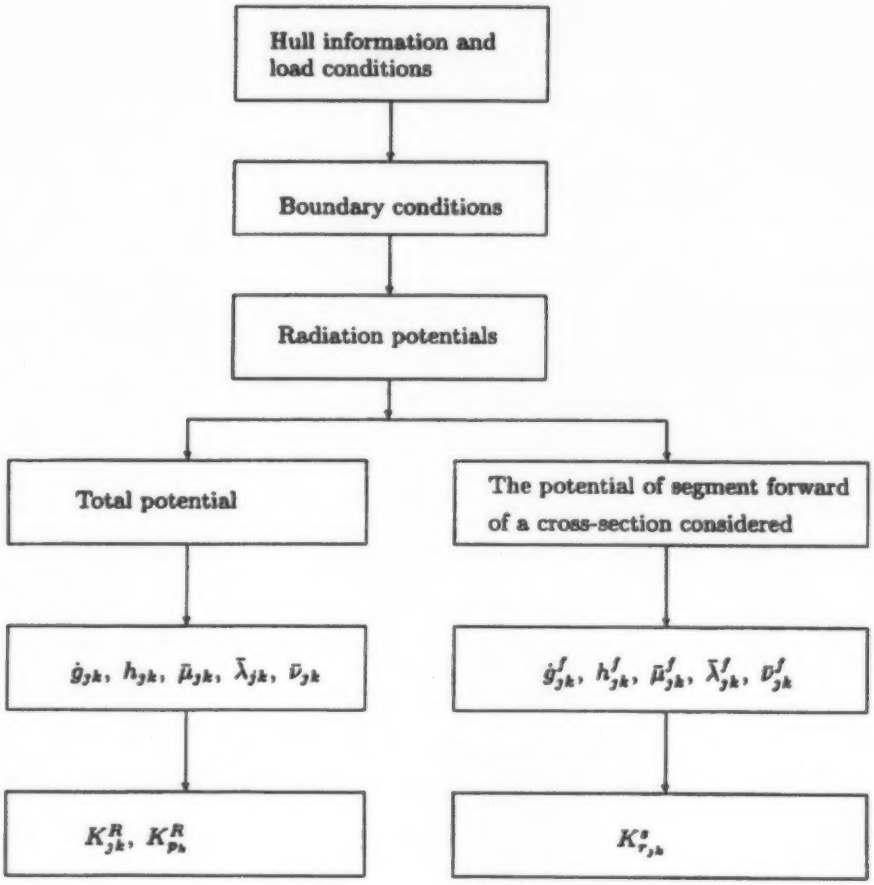

Figure F.2: Code structure of Radiation Module

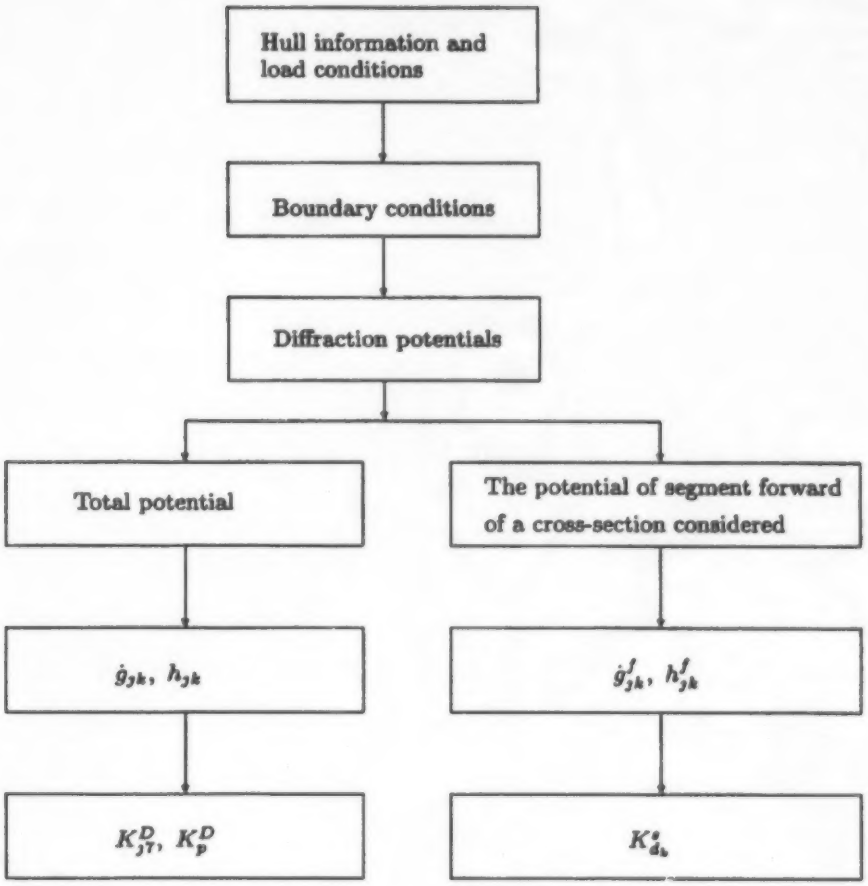

Figure F.3: Code structure of Diffraction Module

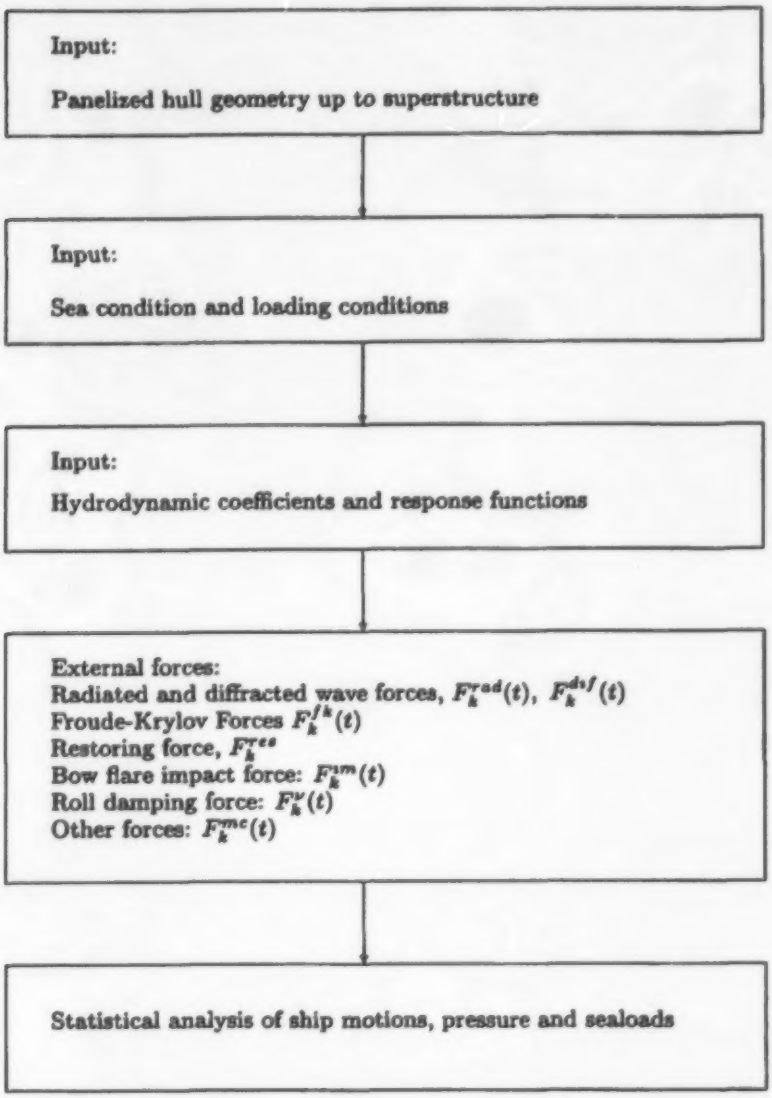

Figure F.4: Code structure of Ship Motion, Pressure and Sea Loads Module

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This report presents the theoretical background for the computer code SEALOADS version 1.0, which computes nonlinear ship motions, sea loads and hydrodynamic pressure distributions over the wetted hull surface in the time domain. The external forces acting on the ship hull include the linearized radiation and diffraction forces, nonlinear Froude-Krylov force, viscous roll damping force, bow-flare slamming force and other forces from rudder, maneuvering, etc. The linearized radiation and diffraction forces are obtained from the impulse response functions, which are solved by using the time-domain Green function and applying a non-impulsive input to the ship. The nonlinear Froude-Krylov force is computed at the instantaneous wetted surface of the hull. A two-dimensional wedge model is applied to compute the bow-flare slamming force. The nonlinear equations of ship motion are solved in the time domain by the fourth-order Runge-Kutta technique.

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